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COMMITMENT TO EQUITY

CEQ INSTITUTE
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Tulane University

Working Paper 87
March 2019

The CEQ Working Paper Series

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ABSTRACT

We analyze the public provision of public goods and income redistribution in a median voter framework. We review existing related frameworks, devoting special attention to their implications for applied analysis. Motivated by empirical regularities discovered in the analysis of CEQ data linking the percentage of "net receivers" to the levels of public good provision, we present an extension of the classic framework of taxation and public good provision that departs from the assumption of a simple proportional tax in favor of a flat tax with exemption. Adjusting the exemption level, we capture tax schemes that are restricted to generating different numbers of net receivers. We then let voters decide on the tax rate and the quantity of public good provided. We find that, in a standard framework, the public good level can *increase or decrease* in the proportion of "net receivers," according to the relationship that exists between the income of the decisive voter and average income in the population conditional on income being larger than the exemption. This suggests that, to account for the richness of comparative statics we observe, the framework used should encompass additional considerations such as turnout and the presence of substitutes for government provided public goods.

JEL Codes: H21, D31, D72

Keywords: public good provision, income distribution, median voter, flat tax, exemption levels.

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On the Political Economy of Redistribution and Public Good Provision

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Abstract

We analyze the public provision of public goods and income redistribution in a median voter framework. We review existing related frameworks, devoting special attention to their implications for applied analysis. Motivated by empirical regularities discovered in the analysis of CEQ data linking the percentage of "net receivers" to the levels of public good provision, we present an extension of the classic framework of taxation and public good provision that departs from the assumption of a simple proportional tax in favor of a flat tax with exemption. Adjusting the exemption level, we capture tax schemes that are restricted to generating different numbers of net receivers. We then let voters decide on the tax rate and the quantity of public good provided. We find that, in a standard framework, the public good level can *increase or decrease* in the proportion of "net receivers," according to the relationship that exists between the income of the decisive voter and average income in the population conditional on income being larger than the exemption. This suggests that, to account for the richness of comparative statics we observe, the framework used should encompass additional considerations such as turnout and the presence of substitutes for government provided public goods.

Version: March 2019

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1 Introduction

The provision of public goods and services, taxation, and income redistribution are among the most important tasks of governments. Not surprisingly, the public economics and political economy literature dealing with these topics is very extensive, using both normative and positive approaches. While the normative approaches are well discussed for example in Boadway and Keen (2000), here we focus on a positive approach, i.e., one that determines the levels of the variables of interest as a function of political competition.

Even with this restriction, the political economy literature remains too vast to be fully considered here. We will thus further focus our attention on the very prominent strand of the literature stemming from the seminal analysis of taxation and redistribution based on direct democracy and the median voter theorem (see e.g., Downs, 1957, Black, 1958, and Foley 1967). Thus, by and large, we do not analyze models of representative democracy and we remand to the reviews of Borck (2007) and Londregan (2009), and to the excellent textbook of Persson and Tabellini (2000) for contributions covering the role of parties (Alesina 1988), different political institutions (Persson and Tabellini, 2003), interest groups (Dixit and Londregan, 1998), political participation (Benabou, 2000), upward mobility in a dynamic model (Benabou and Ok, 2001), and social preferences for fairness (Galasso, 2003), among other topics.

Among median-voter theorem results, Meltzer and Richard (1981) holds a special place of importance in the development of this literature, along with the contributions of Romer (1975, 1977) and Roberts (1977). In Meltzer and Richard (1981), individuals are faced with a labor/leisure decision. Further, the government uses a proportional tax on labor income to finance lump-sum redistribution. The prediction of the model is that the political equilibrium will be determined by the preferences of the voter with median income, and in particular by the relationship between median and mean income.

While it is hard to overstate the contribution of the analysis of Meltzer and Richard, the subsequent literature pointed out the importance of two assumptions: 1) the fact that redistribution is narrowly defined as being in cash, and not more broadly defined through the government provision of public goods, and 2) the linearity of tax rates. Indeed, we present an elaboration of CEQ data showing that, overall, tax systems are far from being proportional, and that government provided goods are a relevant part of government outlays.

The importance of the possibility that tax revenues are used to finance a public good is clearly demonstrated by the analysis of Lovell (1975) and Kenny (1978), and summarized in the textbook by Laffont (1998). We present this analysis in detail later, but, in summary, here agents have exogenous income, but decide how much public good to ask the government to provide. Again, taxes are linear and the outcome of the model is derived applying the median voter theorem as in Meltzer and Richard (1981). However, the implications of the relationship between inequality and level of public good provision turn out to depend on the form of the utility function, and in particular of the comparison between income elasticity of demand for the public good and its price elasticity. Indeed, it is easy to create examples in which the ratio of median to mean income has no effect on the public good level or precisely the opposite effect of that one might expect by simply applying the Meltzer and Richard's framework without distinguishing cash redistribution and public good provision.

The linearity of tax rates is important for two reasons. From the theoretical point of view, the linearity of tax rates is a key feature because, through the budget constraint of the government, one can transform what is a two dimensional decision problem (the level of the proportional tax t and the level of the public good or of the lump-sum redistribution) into a one-dimensional problem. It is well-known that, for even just two dimensions, one cannot expect a majority voting equilibrium to exist in general (as a consequence of Arrow's impossibility theorem). Therefore, being able to work with a one-dimensional problem solves an important technical issue. And under mild regularity conditions on utility, this problem satisfies either single-peakedness or the single-crossing condition (Gans and Smart, 1996), and therefore the median voter theorem applies. The second important consequence of assuming linear tax rates is that the level of government expenditures does not mechanically effect a change in Gini coefficients of post-tax income. This is not otherwise the case, in general. We demonstrate this with analytical derivations based on Lambert (2001) and simple examples that show how the mechanical effect on the Gini coefficients is complex, and it depends both on the form of the tax system, and on the initial form of the income distribution.

An extension of the Meltzer and Richard (1981) model to quadratic tax rates appears in Cukierman and Meltzer (1991), under conditions that allow for a majority voting equilibrium to exist. Another interesting departure from linear tax rates in a taxation model with fixed total revenue target is the majority voting analysis of a flat tax with exemption in Gouveia and Oliver (1996). However, we are not aware of a treatment of a public good model using a flat tax with exemption.

This kind of model appears to be important for a variety of reasons. First, this framework allows for a very immediate way to answer an important question: what are the consequences on the provision of public good if we restrict the tax system in order to protect from taxation the poorest agents? In other words, if we increase the number of "net receivers," are there any unintended consequences on the provision level of public goods? Second, while remaining very clear that one should not interpret our results as causation, but simply as correlational motivating observations, we provide an analysis of CEQ data that shows the existence of relationships between the proportion of net receivers and the level of public goods provided. Third, the applicability of a flat tax with exemption goes well beyond the country level, and it appears to be a staple of local-level taxation in the US.

Our analysis yields two main results. First, we obtain an expansion of the range of utility functions for which public good provision increases with an increase in inequality, with respect to the analysis in Lovell (1975) and Kenny (1978). Second, we find that the public good level can *increase or decrease* in the proportion of "net receivers," according to the relationship that exists between the income of the decisive voter and average income in the population conditional on income being larger than the exemption. This suggests that, to account for the richness of comparative statics we observe, the framework used should encompass additional considerations such as turnout (e.g., Larcinese 2007) and the presence of substitutes for government provided public goods, as in the "ends against the middle" framework of Epple and Romano (1996). (For a complementary result, see the description of "Director's law," in Stigler, 2000.)

The rest of the paper proceeds as follows. First, we present a summary and derive the main implications of the pure redistribution model of Meltzer and Richard (1981). Second, we describe empirical regularities from CEQ data showing that tax rates are not linear and that the public provision of public (or semi-public) goods is important. Further, in a flat tax with exemption regime, we formally present calculations and provide examples that show that there exists a relationship between tax rates, exemption amounts, and reduction in inequality of post-tax incomes. Third, we present a summary and derive the main implications of the public good provision model of Lovell (1975) and Kenny (1978). Then, we present a picture analysis of CEQ data that appears to show a negative correlation between the proportion of "net receivers" and the amount of public good provided for some public goods, and a positive correlation for others. We use this as motivation and introduction for our theoretical results on a flat tax with exemption. We show that an increase in the exemption level can increase or decrease the amount of public good provided, according to the relative position of the income of the decisive voter and the mean income of the population, conditional on being above the exemption. It is worth pointing out that our analysis firmly remains in a one-dimensional framework, because we take the size of the exemption as given. We also consider the consequences of considering turnout and of the existence of substitutes for public provision. Finally, we briefly review papers that tackle the issue of taxation and redistribution without preventively assuming functional forms in a representative democracy framework.

2 The Meltzer and Richard pure redistribution model

Meltzer and Richard (1981) define government size as the share of total income to be redistributed and build a model to explain it as the outcome of the utility maximizing choices of fully informed rational agents. The decisive voter's preferred tax rate maximizes her utility, under a balanced government's budget. The decisive voter thus determines the size of the income redistribution and the size of the government through the political process, referred as the voting rule. Under majority rule, the voter with median income (i.e., the median voter) is the decisive voter. The position of the decisive voter in the income distribution relative to the individual with average income plays an important role in determining the preferred tax rate. Out of this relation emerges the very well-known implication of the model that the higher the inequality (i.e. the ratio of the average income to median income), the higher the size of the redistribution (i.e. the tax rate). Because of its importance, we now describe the Meltzer and Richard model in detail.

All agents share the same utility function, $U(c, l)$, where c denotes consumption and l leisure. Utility is assumed to be strictly concave, and both consumption and leisure are normal goods. Agents differ in their level of productivity, x, in producing consumption goods and decide their labor supply $n = 1 - l$ accordingly. The pre-tax income of an individual with productivity level x is given by

$$y(x) = x\, n(x).$$

The government runs a balanced budget and uses a linear tax schedule to finance lump-sum transfers as its only task. Here, there is no public good provision. Each individual pays a fraction of his/her earned income in taxes, ty, and receives a fixed transfer r. Savings are assumed to be zero. The disposable income after taxes and transfers then becomes

$$c(x) = (1 - t)nx + r.$$

Each agent solves the following optimal labor supply problem by taking t and r as given:

$$\max_{n\in[0,1]} U(c, l) = \max_{n\in[0,1]} U[r + nx(1 - t), 1 - n].$$

The solution of the maximization problem provides the optimal choice of labor supply, $n^*(r, x(1 - t))$ and the critical value of productivity x_0 below which an individual chooses not to work (i.e., for $x \leq x_0, n = 0$ is the optimal choice).

Indeed, the first order condition for the above displayed problem is

$$\frac{\partial U}{\partial n} = U_c[r + nx(1 - t), 1 - n]x(1 - t) - U_l[r + nx(1 - t), 1 - n] = 0.$$

Using concavity of the utility function and $n = 0$, one can establish that optimal labor supply is increasing in x around $n = 0$. Therefore, the agents with $n = 0$, i.e., the individuals who choose not to work, all have $x \leq x_0$, where the critical productivity x_0 solves the first-order condition with $n = 0$. We then obtain

$$U_c(r, 1)x_0(1 - t) = U_l(r, 1),$$

so that the value of x_0 is given by

$$x_0 = \frac{U_l(r, 1)}{U_c(r, 1)(1 - t)}.$$

Per capita income is given by,

$$\bar{y} = \int_{x_0}^{\infty} x\, n(r, (1-t)x) dF(x),$$

where F represents the cumulative distribution of the productivity among agents. This implies that total and per capita income are functions of the values of x_0, t and r. Furthermore, since x_0 depends only on t and r, the government's budget constraint implies that there is a unique amount of transfer consistent with each tax rate:

$$t\bar{y} = r.$$

This is because of the assumption that leisure is a normal good. Indeed, if we increase r, the right-hand side of the above displayed equation increases, while the left-hand side decreases, given that labor supply decreases in r. Therefore, once either t or r is given, the other one can be determined, along with consumption, labor choice, and the size of the government.

At the voting stage, the decisive voter maximizes their indirect utility, taking into account the optimal labor supply choice, by choosing their preferred tax rate. Here, to apply the median voter theorem, Meltzer and Richard (1981) use a result by Roberts (1977) showing that, if the ordering of individual incomes is independent of the choices of t and r, then the individual choice of the tax rate is inversely ordered by income. Meltzer and Richard (1981) accomplish this with two technical results, one showing that consumption increases in government transfers and one showing the ranking of pre-tax incomes is the same of that of productivities. (These are more technical derivations, and we include them in Appendix A.1.)

Given these technical results, it turns out that the problem boils down to maximizing the disposable income of the decisive voter, $y_d(1-t) + r$, again given the optimal labor supply choice. The decisive voter's preferred tax rate maximizes the decisive voter's utility, taking into account the government's balanced budget:

$$\max_{t \in [0,1]} y_d(1-t) + r = \max_{t \in [0,1]} y_d(1-t) + t\bar{y}.$$

The first order condition follows,

$$-y_d + \bar{y} + t\frac{d\bar{y}}{dt} = 0,$$

where $\bar{y}$ is a function of $\tau := (1-t)$ and r, as described above. In what follows, we let $\bar{y}_r$ and $\bar{y}_\tau$ indicate partial derivatives with respect to r and τ. Rewriting the government's budget as $t\bar{y}(\tau, r) = r$ and totally differentiating it, one obtains

$$\{\bar{y} + t[\bar{y}_r y + \bar{y}_\tau(-1) - \bar{y}]\}dt + (t-t)dy + (t\bar{y}_r - 1)dr = 0,$$

and eventually

$$\frac{d\bar{y}}{dt} = \frac{\bar{y}_r \bar{y} - \bar{y}_\tau}{1 - t\bar{y}_r}.$$

Substituting $\frac{d\bar{y}}{dt}$ into the first order condition of the decisive voter's problem, $-y_d + \bar{y} + t\frac{d\bar{y}}{dt} = 0$, we obtain

$$-y_d + ty_d\bar{y}_r + \bar{y} - t\bar{y}\bar{y}_r + t\bar{y}\bar{y}_r - t\bar{y}_\tau = 0.$$

This can be rewritten as an equation that defines the equilibrium tax rate as

$$t = \frac{m - 1 + \eta(\bar{y}, r)}{m - 1 + \eta(\bar{y}, r) + m\eta(\bar{y}, \tau)},$$

where $m = \frac{\bar{y}}{y_d}$ is the ratio of average to median income and $\eta(\bar{y}, r)$ and $\eta(\bar{y}, \tau)$ are partial elasticities with respect to r and τ, respectively (e.g., $\bar{y}_r = \eta(\bar{y}, r)\frac{\bar{y}}{r}$).

Under the common assumption of constant elasticities, the following derivative displays how the ratio of the average income to the income of the decisive voter affects the tax rate.

$$\frac{dt}{dm} = \frac{\eta(\bar{y}, \tau)(1 - \eta(\bar{y}, r))}{(m - 1 + \eta(\bar{y}, r) + m\eta(\bar{y}, \tau))^2} > 0.$$

Therefore, as the ratio of the average income to decisive voter's income increases (i.e., as the level of inequality increases), so do the tax rate, the share of income to be redistributed and hence the size of the government. [1] A review of the empirical evidence in Larcinese (2007), however, shows very limited support for this implication.

2.1 Discussion of the assumptions of linear tax rates and importance of public provision.

The details of the previous analysis of the Meltzer and Richard (1981) model clearly show the importance of assuming that tax rates are linear. This assumption, at least in terms of direct taxes as percentage of pre-tax market income, does not appear to be satisfied for a set of 25 countries for which we obtained CEQ data. Indeed, Table 1 in the next page clearly shows a market increase in the percentage of pre-tax market income paid by the top 2-3 income deciles.

[1] For technical derivations of the Meltzer and Richard Model, please see Appendix A.1.

Table 1. Direct Taxes paid as a Percentage of Market (Pre-tax) Income

DECILE	ARG	ARM	BRA	CHL	COL	CRI	DOM	ECU	SLV	ETH	GHA	GTM	HND	IRN	JOR	MEX	NIC	PER	RUS	ZAF	LKA	TZA	TUN	UGA	URY	VEN
1	0.38	0.16	0.54	-	0.05	3.13	0.00	0.00	-	1.38	0.32	0.01	-	0.61	0.03	0.03	-	-	2.81	0.03	0.00	0.00	0.80	0.00	0.44	-
2	0.31	0.46	0.83	0.00	0.02	4.20	0.01	0.01	0.00	1.32	0.99	0.04	-	1.09	0.03	0.33	0.03	-	4.10	0.14	0.01	0.00	1.52	0.01	1.02	0.00
3	0.29	1.33	1.06	0.00	0.02	4.96	0.02	0.01	0.01	1.26	0.93	0.04	-	1.48	0.03	0.52	0.11	-	4.44	0.31	0.01	0.12	1.69	0.02	1.57	0.00
4	0.24	2.02	1.05	0.01	0.02	4.61	0.02	0.00	0.12	1.25	1.19	0.05	0.02	2.38	0.03	1.02	0.03	-	4.43	0.48	0.01	0.04	3.35	0.01	2.04	0.00
5	0.25	2.56	1.30	0.03	0.03	4.19	0.02	0.01	0.21	1.32	1.42	0.07	-	2.96	0.02	1.39	0.14	-	4.82	0.77	0.03	0.22	4.16	0.19	2.42	0.00
6	0.23	3.01	1.52	0.05	0.03	4.72	0.02	0.04	0.34	1.48	2.07	0.13	0.02	3.29	0.10	1.70	0.17	0.06	6.08	1.36	0.05	0.48	5.02	0.24	2.92	0.01
7	0.21	3.12	1.50	0.14	0.05	5.47	0.06	0.04	0.49	1.41	2.73	0.13	0.04	3.26	0.12	2.16	0.77	0.14	6.59	2.68	0.05	0.71	6.14	0.36	3.69	0.02
8	0.42	4.04	1.93	0.27	0.05	5.77	0.10	0.07	0.72	1.62	3.41	0.14	0.15	3.34	0.19	3.12	1.53	0.16	7.66	5.00	0.10	1.36	7.72	0.74	4.62	0.06
9	1.88	5.00	2.69	0.66	0.09	6.31	0.53	0.16	1.34	2.26	4.25	0.27	0.31	3.55	0.32	3.87	4.07	0.49	8.20	9.91	0.22	2.16	9.22	1.34	6.18	0.09
10	10.94	7.05	7.04	5.31	0.41	8.52	2.99	1.09	2.75	5.35	8.47	1.05	0.95	3.32	4.25	7.63	7.68	3.35	8.75	19.80	1.30	12.86	11.80	6.65	9.85	1.99

Notes: The country codes follow; ARG (Argentina), ARM (Armenia), BRA (Brazil), CHL (Chile), COL (Colombia), CRI (Costa Rica), DOM (Dominican Republic), ECU (Ecuador), SLV (El Salvador), ETH (Ethiopia), GHA (Ghana), GTM (Guatemala), HND (Honduras), IRN (Iran), JOR (Jordan), MEX (Mexico), NIC (Nicaragua), PER (Peru), RUS (Russia), ZAF (South Africa), LKA (Sri Lanka), TZA (Tanzania), TUN (Tunisia), UGA (Uganda), URY (Uruguay), VEN (Venezuela)

Another key assumption is that the government only redistributes cash in this economy. This is a very useful simplifying assumption, but direct government provision is empirically important. Table 2 in the following page shows that direct government provision of goods such as education and health is actually comparable to redistribution amounts.

Table2. Relation between Government Provision of Public Good and Redistributive Measures

COUNTRY	DIRECT TRANSFER (% GDP)	EDUCATION SPENDING (% GDP)	HEALTH SPENDING (% GDP)	DECLINE IN GINI (GINI POINTS)	DECLINE IN GINI (% CHANGE)	NET RECEIVERS (# OF DECILES)
ARGENTINA	5.80	7.48	5.65	6.44	13.56	1
ARMENIA	2.52	3.52	1.68	2.85	7.08	2
BOLIVIA	2.05	8.27	3.60	0.02	0.04	2
BRAZIL	4.07	5.08	4.98	3.54	6.17	2
CHILE	1.64	4.29	3.81	2.95	5.98	2
COLOMBIA	0.46	3.34	4.98	1.16	2.01	6
COSTA RICA	1.18	6.20	6.10	2.20	4.33	1
DOMINICAN REPUBLIC	0.80	3.76	1.84	2.20	4.28	2
ECUADOR	1.22	4.50	3.28	3.28	6.86	6
EL SALVADOR	1.38	2.93	4.29	1.99	4.53	1
ETHIOPIA	1.35	4.62	1.25	1.98	6.15	2
GHANA	0.07	5.65	1.66	1.38	3.15	0
GUATEMALA	0.40	2.71	2.14	0.70	1.37	1
HONDURAS	0.40	6.08	3.53	1.23	2.18	2
INDONESIA	0.39	3.40	0.88	0.76	1.92	10
IRAN	3.99	4.48	2.50	5.73	13.38	9
JORDAN	0.72	3.18	3.10	1.61	4.71	9
MEXICO	0.96	4.54	3.06	2.98	5.84	4
NICARAGUA	0.07	3.80	4.19	1.69	3.52	1
PERU	0.40	2.80	3.10	1.18	2.34	1
RUSSIA	5.28	4.09	3.69	2.77	7.04	1
SOUTH AFRICA	3.81	7.00	4.11	7.66	9.93	5
SRI LANKA	2.00	1.94	1.48	1.15	3.09	3
TANZANIA	0.15	4.60	1.56	4.00	10.47	0
TUNISIA	0.83	6.26	1.59	4.95	11.49	2
UGANDA	0.54	2.40	1.60	1.61	3.90	1
URUGUAY	2.26	3.70	4.68	3.15	6.39	2
VENEZUELA	0.97	4.74	2.75	2.05	5.11	3

Notes: Direct Transfers refer to direct cash and near-cash transfers by the government. Education spending and Health Spending are thought to be to indirect transfers (as referred in CEQ Methodology) and considered as the provision of public good in our models. The decline in Gini coefficient is measured by the difference in Gini coefficient calculated using Market Income (Income before any taxes and transfers) and Consumable Income (Income after taxes and direct transfers and before indirect transfers). This decline refers to the improvement in income inequality as a result of taxes and direct government transfers. An individual or household is a Net Receiver if the direct and indirect taxes paid are less than the direct government transfers received.

The rest of this paper will analyze the consequences of introducing both public goods and non-linear taxation schemes in the economy.

3 The provision of public good using a median voter framework.

We now consider a model of public provision of a public good. A model along these lines appears in Lovell (1975), Kenny (1978) and Laffont (1998). We will see that the basic message of the Meltzer and Richard (1981) can be readily adapted to this framework, but also note important contrasts. Importantly, depending on the relation between income elasticity of demand for the public good and its price elasticity, inequality may increase, leave constant, or decrease the demand of the median voter for public good provision.

We assume that all individuals have the same, standard utility function over consumption c and public good G, $\mathrm{u}(\mathrm{c}, \mathrm{G})$. For the sake of concreteness we adopt a CES formulation, so

$$u(c,G) = \left(c^{-\beta} + G^{-\beta}\right)^{-\frac{1}{\beta}},$$

where $\beta > -1$ is a parameter affecting the elasticity of substitution between private and public good. As it is well known, the restriction $\beta > -1$ makes the utility function strictly quasi-concave. Further, the limit cases of $\beta = -1, 0,$ and $+\infty$ correspond to perfect substitutes, Cobb-Douglas, and perfect complements, respectively. The consumption of agent i depends on her exogenous income $\mathrm{y_i}$ and on the tax set by the government. We begin this section assuming that the government uses a flat tax with rate t. Therefore, the consumption of agent i follows

$$\mathrm{c_i} = (1-\mathrm{t})\mathrm{y_i}.$$

The amount of public good G and the tax rate t are related through the budget constraint of the government. If we denote the total income in the economy with $\overline{\mathrm{Y}}$, we obtain that, to be feasible, G and t must obey

$$\mathrm{G} = \mathrm{t}\overline{\mathrm{Y}}.$$

We define a pair (G, t) as feasible if satisfies the budget constraint displayed above, and if $0 \leq \mathrm{t} \leq 1$ and $0 \leq \mathrm{G} \leq \overline{\mathrm{Y}}$. The amount of public good G and the tax rate t are chosen through the political process. In particular, we look for a feasible pair (G, t) that survives a pairwise majority-voting comparison with any other feasible pair. As in Meltzer and Richard (1981), the government budget constraint allows us to reduce the dimensions of the problem from two to one.

We assume that agents are aware of the government budget constraint and take it into consideration in their choices. Expressing the tax rate as $\mathrm{t} = {}^{\mathrm{G}}/_{\overline{\mathrm{Y}}}$, and substituting into the utility function and the budget constraint, we obtain that the most preferred amount of public good for agent i solves

$$\max_G \mathrm{v_i}(\mathrm{G}) \equiv \mathrm{u}\left(\left(1-{}^{\mathrm{G}}/_{\overline{\mathrm{Y}}}\right)\mathrm{y_i},\mathrm{G}\right) = \left(\left(\left(1-{}^{\mathrm{G}}/_{\overline{\mathrm{Y}}}\right)\mathrm{y_i}\right)^{-\beta} + G^{-\beta}\right)^{-\frac{1}{\beta}},$$

where $\mathrm{v_i}(\mathrm{G})$ is the utility function of this agent expressed only in terms of the public good.

Any interior solution is characterized by the first-order condition, by strict quasi-concavity of the utility function. Assuming an interior solution and defining the most preferred amount of public good by agent i as $\mathrm{G_i}$, the first-order condition $\mathrm{v'_i}(\mathrm{G_i}) = 0$ yields

$$\left(\frac{\mathrm{G_i}}{\overline{\mathrm{Y}} - \mathrm{G_i}}\right)^{1+\beta} = \left(\frac{\mathrm{y_i}}{\overline{\mathrm{Y}}}\right)^{\beta}.$$

Note how the left-hand side of the above displayed equation is increasing in $\mathrm{G_i}$, by $\beta > -1$. However, the right-hand side may be decreasing, constant, or increasing in the ratio $\mathrm{y_i}/\bar{Y}$, depending on whether β is negative, zero, or positive. In any case, the most preferred public good amounts for agents are ordered by income and therefore the median voter theorem applies. If we denote with $\mathrm{y_a}$ the average income in the population, with $\mathrm{y_m}$ the median income, normalize the total population n to 1, and denote with $\mathrm{G_m}$ the most preferred amount by the median-income voter, the political equilibrium results in $\mathrm{G_m}$ as the unique value that solves

$$\left(\frac{\mathrm{G_m}}{y_a - \mathrm{G_m}}\right)^{1+\beta} = \left(\frac{\mathrm{y_m}}{y_a}\right)^{\beta}.$$

While the treatment above is similar to Meltzer and Richard (1981), the comparative statics with respect to changes in median to mean income ratio now depends on β. If $-1 < \beta < 0$, then, for fixed $\mathrm{y_a}$, the right-hand side decreases in $\mathrm{y_m}/y_a$. Therefore, an increase in inequality (measured as a decrease in $\mathrm{y_m}/y_a$, again for fixed $\mathrm{y_a}$) increases the amount of public good provided, just as in Meltzer and Richard (1981). But, if $\beta = 0$, then inequality has no effect on public good provision. And, if $\beta > 0$, then inequality decreases public good provision. Kenny (1978) shows how this result more generally depends on the comparison between income elasticity of demand and its price elasticity. Husted and Kenny (1997) exploit this fact in their econometric analysis on the effect of the expansion of the voting franchise on the size of government.

4 Extension to non-linear tax schemes

Here we maintain a median-voter framework as much as possible. We discuss abandoning this framework in Section 6.

We focus our analysis on the majority voting model of Gouveia and Oliver (1996) of a flat tax with exemption.[2]

4.1 Gouveia and Oliver (1996)

In this model there is only one good, so the utility of agent i is identified with their after-tax income. In turn, after tax income depends on the exogenous income y_i and on the total tax paid, $T(a, y)$. To capture the structure of a flat tax with rate t and exemption level a, we have

$$T(a, y) = \max\{0, t(y - a)\}.$$

Income in the economy is distributed with a cumulative distribution F with density f. Therefore, if we denote the total amount of resources collected by the tax system with $R(t; a)$, we obtain $R(t; a) = t\varphi(a)$, where

$$\varphi(a) = \int_a^\infty (y - a) f(y) dy.$$

Note that

$$\varphi(a) = (1 - F(a)) \int_a^\infty \frac{(y - a)}{1 - F(a)} f(y) dy = (E(y|y \geq a) - a)\big(1 - F(a)\big).$$

It is assumed that the government must raise a total amount of resources equal to an exogenously specified amount $\overline{R}$, which does not enter the utility of the agents. Therefore, the government's budget constraint then requires

$$R(t; a) = \overline{R},$$

and this equation can be used to express the rate t as a function of the exemption level a. Indeed, using this equation, it is possible to calculate that

$$\frac{dt}{da} = \frac{d}{da}\left(\frac{\overline{R}}{\varphi(a)}\right) = -\frac{\overline{R}\varphi'(a)}{\big(\varphi(a)\big)^2} = \frac{t\big(1 - F(a)\big)}{\varphi(a)}.$$

Clearly, not all exemption levels are consistent with $R(t; a) = \overline{R}$. Indeed, if $F(a) = 1$, then no revenue is collected from the tax. So, denote with $\tilde{a}$ the largest possible exemption amount, corresponding to a tax rate of 1.

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[2] Another interesting avenue of departure from linear taxation is Cukierman and Meltzer (1991) who present a model of quadratic taxation and redistribution and characterize conditions that allow for a majority voting equilibrium to exist.

Agents then decide with a majority voting election on the level of the exemption. Clearly, agents with income below a are indifferent to the level of the exemption. As for agents with income above the exemption level, what matters in in their decision is how their post-tax income changes, as increases in the exemption level decrease the tax base and increase the tax rate, by the government's budget constraint. The change in post-tax incomes is given by the following:

$$\frac{d}{da}\big(y - t(y - a)\big) = +t - (y - a)\frac{dt}{da} = -\frac{t}{\varphi(a)}\Big((y - a)\big(1 - F(a)\big) - \varphi(a)\Big)$$

$$= \frac{t\big(1 - F(a)\big)}{\varphi(a)}(E(y|y \geq a) - y).$$

Therefore, we see that agents with income below the unconstrained mean $E(y)$ of the population would prefer to set a as large as possible, i.e., to $\tilde{a}$. Similarly, agents with income above $E(y|y \geq \tilde{a})$ always prefer smaller levels of the exemption. It turns out that the preferences of agents with income between $E(y)$ and $E(y|y \geq \tilde{a})$ do not have single-peaked preferences, but under the assumption that the distribution of income is right-skewed, i.e., $F(E(y)) > 1/2$, the the majority voting equilibrium is to select the largest possible exemption level, and completely expropriate any income in excess of the exemption $\tilde{a}$.

4.2 On the consequences of a flat tax with exemption on changes in inequality.

The conclusion of Gouveia and Oliver (1996) is, in a narrow sense, a common sense one: in a pure endowment economy, i.e., without incentive effects, the poor "fully" expropriate the rich, of course within the limits imposed by the exogenous choice of fiscal instrument. However, the techniques they pioneer prove to be very useful.

In addition, it is worth pointing out that, with a flat tax with exemption regime, the taxation regime itself tends to reduce inequality, measured for example with the Gini coefficient. Following for the rest of this section Lambert (2001), including his notation, one can express the Gini coefficient G as

$$G = \frac{2}{\mu} Cov[y, F(y)],$$

as we demonstrate in the technical derivation at the end of this chapter.[3] Changes in the Gini coefficient that result from different combination of rate t and exemption level a can be calculated only with reference to the whole distribution of income. In the formula above, income y is transformed to y-$T(a, y)$, with $T(a, y) = \max\{0, t(y - a)\}$. Changes in a affect both the numerator, $Cov[y, F(y)]$, and the denominator μ in non-trivial manners.

[3] Please see Appendix A.2. for the technical derivation.

The following toy examples show that, as soon as one abandons the no-exemption case, one should expect changes in the Gini coefficient in going from pre-tax to post-tax income. While increases in t tend to decrease the Gini coefficient (see, e.g., the comparison between Tables 3b and 3c), increases in a first decrease, and then increase the after-tax Gini coefficient (see, e.g., the comparison between Tables 3a, 3b, and 3d).

Table 3a: Toy Example: Exemption Level = 0, Tax Rate 20%

	1	2	3	4	TOTAL	GINI
ORIGINAL INCOME	10	20	30	40	100	0.2500
AFTER TAX INCOME	8	16	24	32	80	0.2500

Table 3b: Toy Example: Exemption Level = 10, Tax Rate 20%

	1	2	3	4	TOTAL	GINI
ORIGINAL INCOME	10	20	30	40	100	0.2500
AFTER TAX INCOME	10	18	26	34	88	0.2273

Table 3c: Toy Example: Exemption Level = 10, Tax Rate 30%

	1	2	3	4	TOTAL	GINI
ORIGINAL INCOME	10	20	30	40	100	0.2500
AFTER TAX INCOME	10	17	24	31	82	0.2134

Table 3d: Toy Example: Exemption Level = 20, Tax Rate 20%

	1	2	3	4	TOTAL	GINI
ORIGINAL INCOME	10	20	30	40	100	0.2500
AFTER TAX INCOME	10	20	28	36	94	0.2287

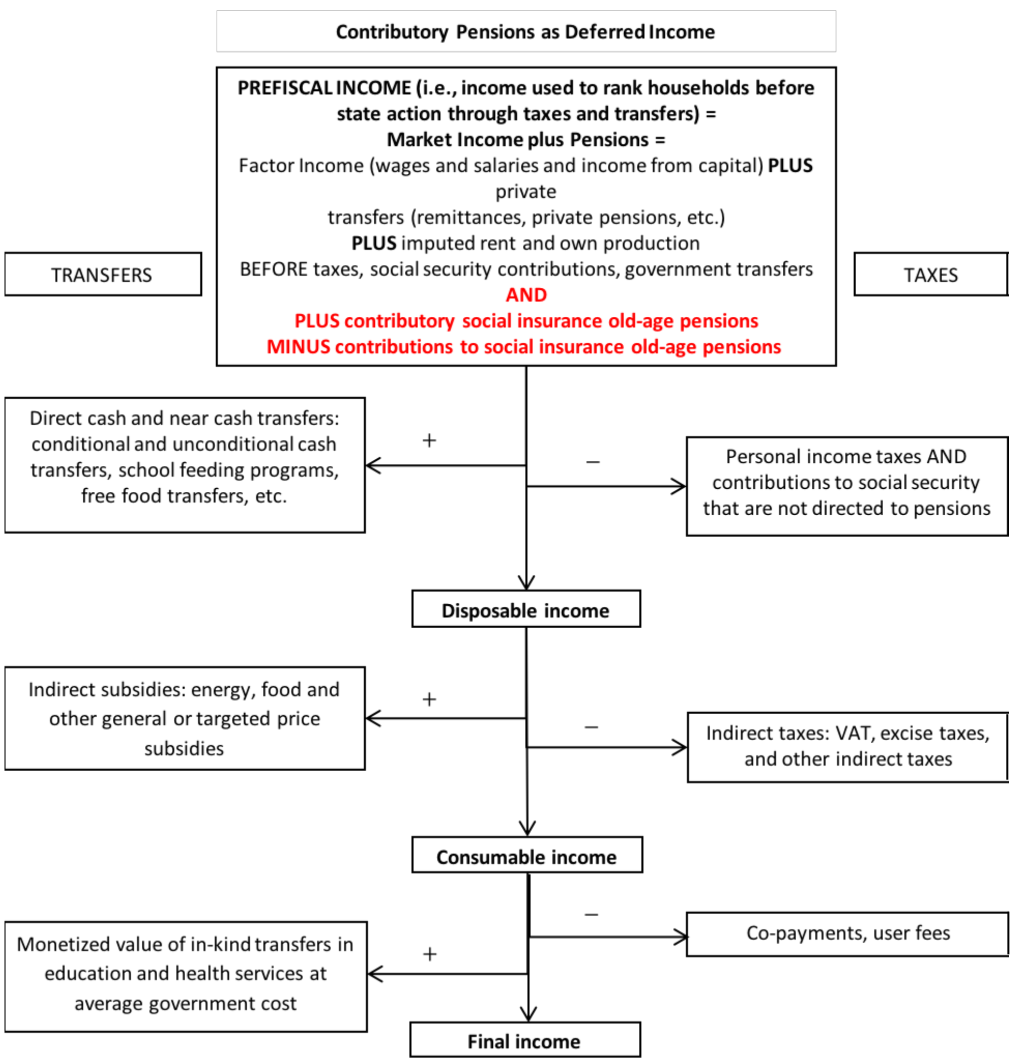
The comparison between Tables 3b and 3c) is also illustrative of another important fact. Suppose that the government needs to raise first 12 (3b) and then 18 (3c), because the size of expenditures has increased. This can be accomplished leaving the exemption level at 10, and raising the tax rate from 20% (3b) to 30% (3c). Since the exemption level is not zero, this results in a reduction of the Gini coefficient for post-tax income from 0.2273 (3b) to 0.2134 (3c). In what follows, we refer to this fact as the "mechanical" reduction in inequality.

5 The provision of public goods financed with a flat tax with exemptions.

5.1 Empirical Patterns

We begin with a description of empirical patterns. We are interested in the relation between fiscal incidence/inequality reduction and the provision of public goods. We are not interested in the relation between initial inequality and redistribution or provision of public goods, in contrast with the majority of the empirical analysis conducted so far. Of course, our description has no pretense of establishing any kind of causation relation. We just want to illustrate the kind of patterns that may arise in the real world. The data used in the figures are from CEQ Fiscal Incident Assessment Analyses for various countries and World Bank Development Indicators.

CEQ Analysis uses the following income concepts to measure fiscal incidence.



In the figures below, fiscal incidence is represented by the number of deciles of net receivers, i.e. those with a higher consumable income relative to the market income. Inequality reduction is given by the decline of the Gini coefficient from market income to consumable income. Education and health spending as a percentage of GDP represent the provision of public good.

One common trend in the figures is that the provision of the public good declines with the number of deciles as net receivers, while it increases with the reduction in inequality measured by the decrease in Gini coefficient. (The relationship between provision of public good and reduction in inequality is unaffected by the way we measure the decrease in Gini coefficient, namely in Gini points or percentage change). One exception to this trend is tertiary education spending which is measured by per student spending as a percentage of per capita income.

The fact that more spending on public goods is positively correlated with a decrease in the Gini coefficient is easily interpreted in light of our discussion following the examples we present in Section 4.3. Given the constraint of working with a the flat tax with exemption and keeping the exemption level fixed, increases in public good expenditures must result in a larger tax rate, and therefore in a more progressive overall taxation scheme.

Therefore, in the model that follows the presentation of the empirical evidence, we focus on the effect of the number of deciles that are net receivers. In the model, this number is identified as the exemption level in a flat-tax-with-exemption taxation scheme, as in Gouveia and Oliver (1996).

Figure 1.A – Education Spending (% of GDP) and Number of Deciles as Net Receivers

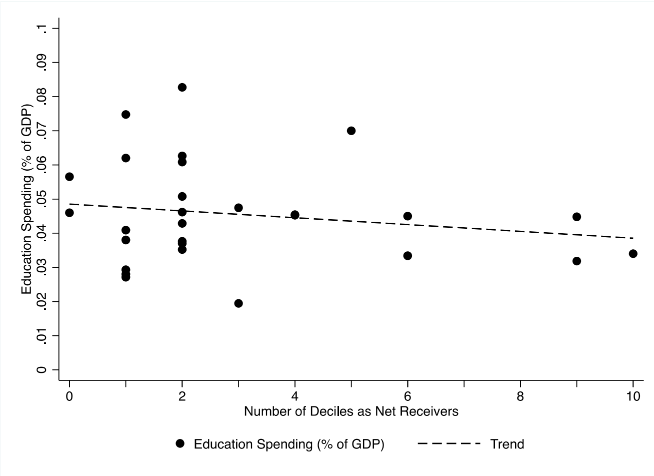


Figure 1.B – Education Spending (% of GDP) and Decline in Gini Coefficient

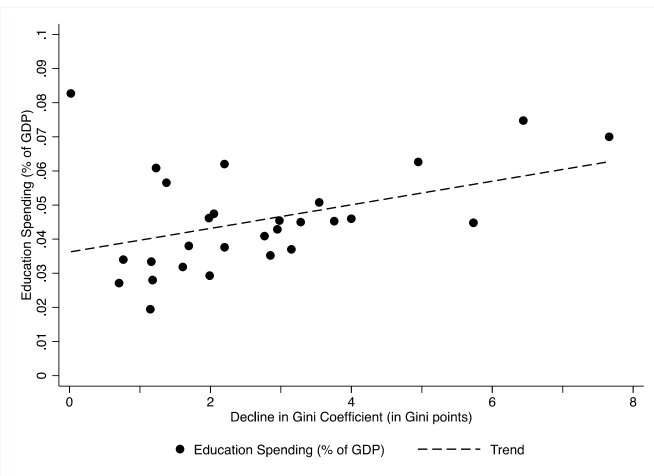


Figure 2.A – Health Spending (% of GDP) and Number of Deciles as Net Receivers

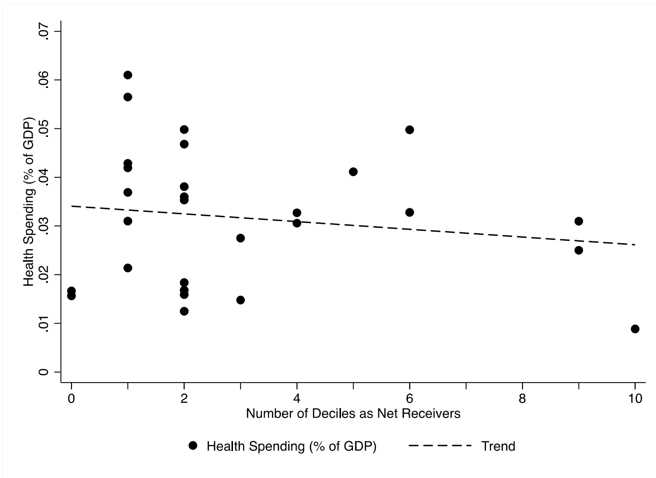


Figure 2.B – Health Spending (% of GDP) and Decline in Gini Coefficient

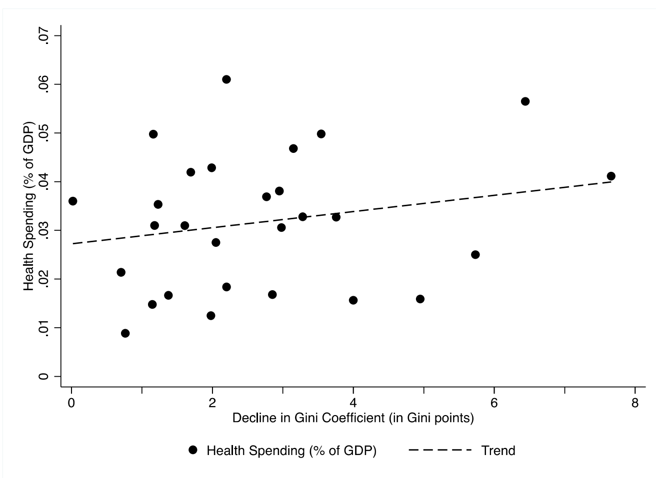


Figure 3.A – Primary and Secondary Education Spending (% of per capita GDP) and Number of Deciles as Net Receivers

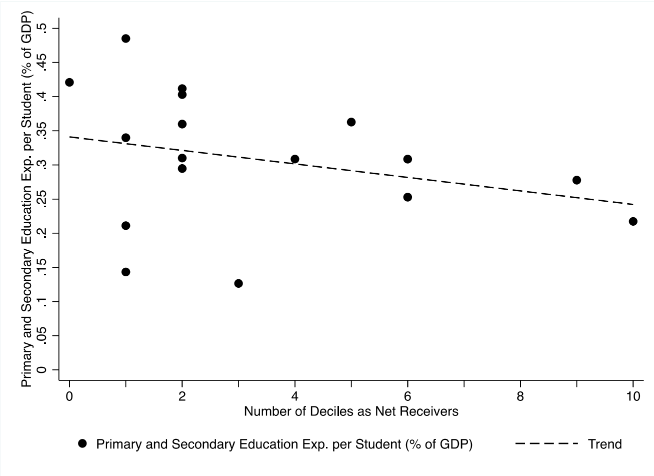


Figure 3.B – Primary and Secondary Education Spending (% of per capita GDP) and Decline in Gini Coefficient

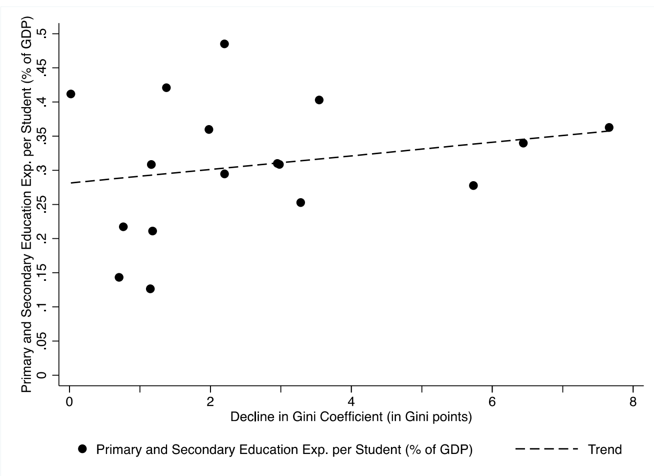


Table3. Correlations between Indirect Transfers (Provision of Public Good) and Redistributive Measures

	Dependent Variable		
	Total Education Spending as a percentage of GDP	Health Spending as a percentage of GDP	Primary and Secondary Education Spending as a percentage of per capita GDP
Panel A			
Decline in Gini in Gini points	0.003	0.000	0.004
	(0.002)	(0.001)	(0.008)
Constant	0.038***	0.020***	0.298***
	(0.009)	(0.005)	(0.054)
Panel B			
Decline in Gini in % Change	0.001	-0.000	-0.000
	(0.001)	(0.001)	(0.005)
Constant	0.038***	0.022***	0.302***
	(0.009)	(0.005)	(0.055)
Panel C			
Net Receivers in # of Deciles	-0.00131**	-0.00127**	-0.0133**
	(0.00616)	(0.00725)	(0.0052)
Constant	0.045***	0.024***	0.352***
	(0.007)	(0.005)	(0.058)

Notes: Each panel represents a set of regressions of a particular indirect transfer (education or health) on a redistributive measure as an independent variable, for example the decline in Gini coefficient in Gini points. The outlier countries are excluded (i.e. South Africa in total education, Colombia in total health spending, and Guatemala in primary and secondary education spending) to have a better representation of the associations. The data regarding Total Education Spending as a percentage of GDP and Health Spending as a percentage of GDP are taken from CEQ Indicators, whereas Primary and Secondary Education Spending as a percentage of GDP per capita is taken from World Bank Indicators. Robust standard errors are in parentheses. * p<0.10, ** p<0.05, *** p<0.01.

5.2 Theoretical Model

The framework we use combines the previous public good model with the flat tax with exemption studied in Gouveia and Oliver (1996), so our tax scheme imposes average progressivity in the tax code. As before, we assume that all individuals have the same utility function over consumption c and public good G, $\mathrm{u}(\mathrm{c},\mathrm{G})$. For the sake of concreteness, we adopt the same constant elasticity of substitution formulation in the public good adopted earlier, so

$$u(c,G)=\left(c^{-\beta}+G^{-\beta}\right)^{-\frac{1}{\beta}}.$$

The consumption of agent i depends on their exogenous income $\mathrm{y_i}$ and on the tax set by the government. The difference with previous formulations is that now we consider the consequences of a government that uses a flat tax with rate t *and* exemption level a. In this case, the tax paid by agent i is zero if their income is below a, while it equals $\mathrm{t}(\mathrm{y_i}-a)$ if $\mathrm{y_i}\geq a$. Therefore, the consumption of agent i is $\mathrm{c_i}=\mathrm{y_i}$ if $\mathrm{y_i}<a$, otherwise it becomes $c_i=(1-\mathrm{t})\mathrm{y_i}+ta$. For simplicity, we conduct the analysis in a continuum of agents framework, so we can normalize the size of the population n to 1, and we describe the distribution of income in the economy with a cumulative distribution F with density f. The crucial assumption we make is that a is exogenous. This could be a restriction that arises for normative reasons on the number of agents that are net receivers. We then let the political process determine the level of the tax rate and of the public good. As in the previous formulation of the model, we can use the government's budget constraint to reduce the dimensionality of the problem. If we denote the total amount of resources collected by the tax system with $\mathrm{R}(\mathrm{t};\mathrm{a})$, we obtain $\mathrm{R}(\mathrm{t};\mathrm{a})=\mathrm{t}\varphi(\mathrm{a})$, where

$$\varphi(\mathrm{a})=\int_a^\infty (y-a)f(y)dy.$$

The government's budget constraint then requires $\mathrm{R}(\mathrm{t};\mathrm{a})=G$. Solving this equation for the implied tax rate given the exemption and public good levels, we obtain

$$\mathrm{t}=\frac{G}{\varphi(\mathrm{a})}.$$

We are now ready to find the optimal level of public good desired by agent $\mathrm{y_i}$. If $\mathrm{y_i}<a$, then this agents desires largest possible level of the public good. This level is found where $\mathrm{G}=\varphi(\mathrm{a})$, or, in other words, for a tax rate that equals 1. This will be the equilibrium if the exemption level is so large that $\mathrm{F}(a)\geq\frac{1}{2}$. In this case we observe the complete expropriation of all income above a minimum level a, with all the tax receipts devoted to the provision of public goods. In the more interesting case where $\mathrm{F}(a)<\frac{1}{2}$, we can substitute the value of $\mathrm{t}=G/\varphi(\mathrm{a})$ into the consumption level of agent i with income $\mathrm{y_i}\geq a$ to obtain

$$c_i=\left(1-\frac{G}{\varphi(\mathrm{a})}\right)\mathrm{y_i}+\frac{G}{\varphi(\mathrm{a})}a=\mathrm{y_i}-\frac{\mathrm{y_i}-a}{\varphi(\mathrm{a})}G.$$

Thus, the most preferred amount of public good for agent i solves

$$\max_G v_i(G) \equiv u\left(y_i - \frac{y_i - a}{\varphi(a)} G, G\right).$$

Assuming again an interior solution and defining the most preferred amount of public good by agent i as G_i, the first-order condition $v'_i(G_i) = 0$ yields

$$\frac{y_i - a}{\varphi(a)} = \left(\frac{y_i - G_i \frac{y_i - a}{\varphi(a)}}{G_i}\right)^{1+\beta}.$$

Clearly, this is a more complex equation than what we obtained for the linear tax case. But we can easily observe that changes immediately emerge for particular parameter configurations. For example, if we set $\beta = 0$ we recover the Cobb-Douglas case, but now this results in

$$G_i = \frac{\varphi(a)}{2} \frac{y_i}{y_i - a}.$$

In the model without exemption, Kenny (1978) shows that for Cobb-Douglas utility, the desired public good level is independent of income, if taxation is proportional. Indeed, we can recover that result by setting a to zero in the above displayed equation and noticing that income cancels out. However, as soon as $a > 0$, the above displayed expression decreases in individual income; furthermore, one can show that the desired public good level is decreasing in income for any $\beta \leq 0$. Therefore, by continuity, there must exist some $\bar{\beta}>0$ such that, if $\beta < \bar{\beta}$, then the desired public good level is decreasing in income. In other words, as compared to proportional taxation, a flat tax rate with exemption makes it more likely that the desired public good level is decreasing in income for agents with income above the exemption.

This is important because, as all agents with income below the exemption always vote for the largest public good amount in a pairwise election, if the desired public good level is decreasing in income for incomes above the exemption as well, then the median voter theorem applies. Otherwise, the situation is not so clear, and we may have the case of a coalition of poor and rich agents who want to spend large amounts on public goods, while the middle class wants to reduce public good spending. This bears similarities, but also differences, with the "ends against the middle" result of Epple and Romano (1996a), discussed below.

In general, however, we see that the public good level now does not simply depend on the ratio of median to mean income, but on a more complex formula that relates $\beta, y_m, a,$ and $\varphi(a)$. To understand better these relationships in light of our future analysis, it is worth pointing out, as Gouveia and Oliver (1995) do, that

$$\varphi(\mathrm{a}) = (1 - F(a)) \int_a^\infty \frac{(y-a)}{1-F(a)} f(y)dy = (E(y|y \geq a) - a)(1 - F(a)),$$

i.e., a relationship between mean income conditional on income being larger than the exemption, the value of the exemption itself, and the proportion of agents whose income is above the exemption.

Another interesting question regards the behavior of the equilibrium spending on public goods as the exemption level is raised. Assuming that the median voter theorem applies and recalling our first-order condition, we have

$$\frac{\mathrm{y_m} - a}{\varphi(\mathrm{a})} = \left(\frac{\mathrm{y_m} - \mathrm{G_m} \frac{\mathrm{y_m} - a}{\varphi(\mathrm{a})}}{\mathrm{G_m}} \right)^{1+\beta}.$$

Now applying the implicit function theorem, we see that the sign of the change in $\mathrm{G_m}$with respect to a has the same sign of that of $\varphi(\mathrm{a})/(\mathrm{y_m} - a)$. We then obtain

$$\frac{d}{da}\left(\frac{\varphi(\mathrm{a})}{\mathrm{y_m} - a}\right) = \frac{\varphi'(\mathrm{a})(\mathrm{y_m} - a) + \varphi(\mathrm{a})}{(\mathrm{y_m} - a)^2}.$$

Since $\varphi'(\mathrm{a}) = -\big(1 - F(a)\big)$, the numerator of the above displayed equation can be simplified as

$$\begin{aligned} &\varphi'(\mathrm{a})(\mathrm{y_m} - a) + \varphi(\mathrm{a}) = \big(1 - F(a)\big)(E(y|y \geq a) - a - \mathrm{y_m} + a) \\ &= \big(1 - F(a)\big)(E(y|y \geq a) - \mathrm{y_m}). \end{aligned}$$

So, if the distribution of income is right-skewed, so that the average income is above the median, increasing the exemption level *increases* the amount of public good provided in the political equilibrium.

Indeed, this is can be intuitively explained in a different way. Recall that agents are maximizing

$$\mathrm{u}\left(\mathrm{y_i} - \frac{\mathrm{y_i} - a}{\varphi(\mathrm{a})} G, G\right),$$

so we see that $\frac{\mathrm{y_i}-a}{\varphi(\mathrm{a})}$ acts as a public good price. Since this price decreases with a, if G is a normal good, then agents will demand more G.

A first interesting caveat is offered by Boix (2003), who points out that turnout should be included in considerations such as ours (see also Larcinese, 2007). Boix (2003) emphasizes the importance of turnout in determining the size of the public sector, hence the level of redistribution. In his model, as turnout among the least skilled worker declines, the position of the median voter gets closer to the position of the voter with average income, which results in a smaller public sector.

In our model, what matters for electoral success is the income of the median *voter*, and it is well known that turnout is increasing in income (see, e.g., Wolfinger and Rosenstone, 1980). We consider a scenario where the median voter is positioned above the voter with average income because of turnout. In the presence of such a turnout pattern, where the median income *among voters* is above the average of the overall population income, the sign of $\frac{d}{da}\left(\frac{\varphi(\mathrm{a})}{\mathrm{y_m}-a}\right)$ reverses, and we obtain the opposite implication: increasing the exemption level *decreases* the amount of public good provided in the political equilibrium.

A second interesting caveat arises in the presence of private substitutes for the public good, as in the "ends against the middle" framework of Epple and Romano (1996a). (See also Glomm and Ravikumar, 1996; Epple and Romano, 1996b; Gouveia, 1996; Fernandez and Rogerson, 1996; and the exposition of Director's Law in Stigler, 1970.) Epple and Romano (1996a) set up a voting model in which agents can opt out of consuming the public good amount publicly provided, and instead consume an amount that they privately acquire. Clearly, it is sufficiently rich agents that may decide to do so, at a level that is discretely larger than what is publicly provided. There are two interesting possibilities, and they are usefully described according to the analysis of Kenny (1978) of a public good financed with a proportional tax and without available substitutes. If agents' most desired level of publicly provided G is decreasing in income, then no changes arise when a substitute is present. The median voter remains decisive, since richer voters demand less public good, and the possibility of opting out only further reduces their demand. In contrast, if agents' most desired level of publicly provided G increases with income, then the availability of substitutes makes preferences not single-peaked, and equilibrium may fail to exist. When equilibrium does exist, the median voter is not decisive anymore, since very high income agents opt out and pool with very low income ones to support a lower level of public expenditure. Therefore, the decisive voter has income below the median.

Combining this framework with our flat tax with exemption has two effects: first, as we demonstrated in our Cobb-Douglas example, it is now more likely that a utility function displays decreasing most desired levels of public good in income, and so it is more likely that the usual median voter theorem applies. Second, if instead most desired public good levels are increasing in income, then what matters in the comparative statics linking G and a is the sign of $\frac{d}{da}\left(\frac{\varphi(\mathrm{a})}{\mathrm{y_d}-a}\right)$ at some decisive income level $\mathrm{y}_d < \mathrm{y}_m$. Therefore, even considering the effects of turnout, we tend to restore the implication that increasing the exemption level *increases* the amount of public good provided in the political equilibrium. Of course, a definite prediction requires institutional knowledge of the patterns of income, turnout, and the presence of private substitutes for the public good.

6 Taxation and redistribution models without functional form assumptions.

There are at least two strands of the literature that abandon the assumptions of a unidimensional policy space. Both strands have to contend with the problem that equilibrium in pure strategies does not exist. The problem arises because, for any tax/redistribution schedule, given enough flexibility, it is always possible to reduce the tax paid by 50% of the electorate by a small amount, compensating this reduction by expropriating a small fraction of the electorate. The problem is similar to the division of a cake among three agents by majority voting: for any proposed division, it is always possible to find a coalition of two agents that does better by expropriating the third.

A first important strand is the one stemming from the seminal contribution of Myerson (1993) studying the implications of assuming that redistribution is perfectly targetable. These models are not models of direct democracy, but models of representative democracy and tend to admit equilibria only in mixed strategies. Indeed, these equilibria resemble those of all-pay auctions. A very important contribution is that of Lizzeri and Persico (2001). They introduce a model with two competing, office-motivated politicians who can use the resources raised from taxation to either redistribute wealth, or pay for the provision of public goods. The key assumption is that public goods are not easily targetable to a subset of voters; while redistribution can be (this notion of redistribution therefore captures both cash transfers and pork-barrel projects). Indeed, in their model there is no assumption at all on the shape of redistribution, and in particular there is no restriction to linear tax rates and lump-sum redistribution. It is worth pointing out that, in this framework, redistribution does not go from the rich to the poor. Rather, it is determined by the electoral incentive that office-motivated politicians have to aggressively tax a minority and redistribute the amount collected to a majority of voters; so this redistribution ends up increasing inequality. Politicians therefore face a tradeoff: while the benefits from the public good may be higher for society as a whole, pork-barrel projects and cash redistribution can be targeted, and therefore may end up undercutting the efficient choice. The main interest of Lizzeri and Persico (2001) is to compare the performance of different electoral incentives; indeed they find that the under-provision problem is especially severe for winner-take-all systems. However, their equilibrium lends itself to a comparative statics analysis that is relevant in relating changes in inequality brought about by taxation and redistribution, and level of public good provision. Equilibrium is in mixed strategies: with some probability the public good is provided, and with the complementary probability targeted, inequality enhancing redistribution is implemented. Therefore, there arises a positive correlation between observing the provision of public good and a more equitable pattern of cash taxes and transfers. Crutzen and Sahuguet (2009) also contribute to this strand of the literature and consider the possibility that taxation can be targetable, in addition to redistribution. Further, they analyze the effects of distortions in taxes.

A second very important strand of the literature considers the problem of raising a fixed amount of revenue, without imposing restrictions on the form of the tax schedule, or and tries to determine whether a progressive tax scheme emerges in equilibrium. Carbonell-Nicolau and Ok (2007) presents a model of representative democracy with two office-motivated parties who compete against each other by offering tax functions that, in principle, are not restricted to be marginal-rate progressive (i.e., convex), and therefore linear. The only assumption they make is that income distributions are right-skewed, i.e., they assume that the median is below the mean, and they investigate if this assumption is sufficient to generate an equilibrium in marginal-rate progressive tax schedules. Carbonell-Nicolau and Ok show that equilibrium exists, but only in mixed strategies, and, in general it is not possible to exclude that a marginal-rate regressive scheme is chosen in equilibrium with positive probability. This result complements Marhuenda and Ortuno-Ortın (1995), which says that if the median voter's income is below the mean income and voting is self-interested, any marginal rate progressive (convex) tax schedule defeats any marginal rate regressive (concave) one. This shows how it is the possibility of tax schedules that are neither globally convex nor concave that creates voting cycles.

To avoid mixed-strategy equilibria and restore a deterministic prediction, other papers in this strand of the literature introduce additional elements to the political process: uncertainty about abandoning the status-quo as in Marhuenda and Ortuno-Ortın (1998), policy preferences of the candidates and costs of participating in the election as in Carbonell-Nicolau and Klor (2003), internal party dynamics as in Roemer (1999). These models deliver, in equilibrium, tax schedules with increasing marginal taxation rates.

7 Conclusion.

We investigate the relationship between public provision of public goods and income redistribution using a median voter model. The Fiscal Incidence Analyses conducted by Commitment to Equity Institute suggest a negative association between the percentage of tax-exempt individuals ("net receivers" in our analysis) and the level of public provision of public goods. We start our analysis by reviewing the previous literature and then offer an extension of the classical framework of taxation and public good provision that departs from the assumption of a simple proportional tax in favor of a flat tax with exemption. Adjusting the exemption level, we capture tax schemes that are restricted to generating different numbers of net receivers. We then let voters decide on the tax rate and the quantity of public good provided. Our model predicts that the level of public good provision can increase or decrease in the proportion of "net receivers" depending on the relationship between the income of the decisive voter and the average income in the population, conditional on income being larger than the exemption. More specifically, if the distribution of income is right-skewed, so that the average income is above the median, increasing the exemption level *increases* the amount of public good provided in the political equilibrium. We conclude by pointing out two caveats, which may change the direction of the model's prediction. If turnout patterns are such that the median income *among voters* is above the average of the overall population income, then increasing the exemption level *decreases* the amount of public good provided in the political equilibrium as observed in the CEQ Data. Also, the presence of substitutes for government provided public goods may result in agents opting out of consuming the public good amount publicly provided, and instead consuming an amount that they privately acquire. Under certain conditions allowing the existence of an equilibrium, very high-income agents opt out and pool with very low-income ones to support a lower level of public expenditure, hence resulting in a negative relationship between the level of public good provision and exemption level.

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Appendix

A.1. Technical Derivations of Meltzer and Richard (1981) Model

Response of Consumption to Government Transfers

For those who subsist on welfare (i.e. $n = 0$), consumption is equal to the government transfer (i.e. $c = r$).

$\Longrightarrow \frac{\partial c}{\partial r} = 1$ for individuals who choose not to work.

For those work, recall that;

$c = r + nx(1 - t)$

Differentiating with respect to r,

$$\Longrightarrow \frac{\partial c}{\partial r} = 1 + \frac{\partial n}{\partial r}\, x(1 - t)$$

$$\Longrightarrow \frac{\partial n}{\partial r} = \left(\frac{\partial c}{\partial r} - 1\right)\frac{1}{x(1-t)}$$

Also, the second order condition of the maximization problem follows,

$$\frac{\partial^2 U}{\partial n^2} = \{U_{cc}x(1 - t) - U_{cl}\}x(1 - t) - U_{lc}x(1 - t) + U_{ll}$$

$$= U_{cc}x^2(1 - t)^2 - 2\, U_{cl}x(1 - t) + U_{ll} \equiv D < 0 \text{ since } U\,(c, l) \text{ is strictly concave.}$$

Now, differentiate the first order condition with respect to r,

$$\left\{U_{cc}\frac{\partial c}{\partial r} - U_{cl}\frac{\partial n}{\partial r}\right\}x(1-t) = U_{cl}\frac{\partial c}{\partial r} - U_{ll}\frac{\partial n}{\partial r}$$

$$\Rightarrow U_{cc}\frac{\partial c}{\partial r}x(1-t) - U_{cl}\frac{\partial n}{\partial r}x(1-t) = U_{cl}\frac{\partial c}{\partial r} - U_{ll}\frac{\partial n}{\partial r}$$

$$\Rightarrow \{U_{cc}x(1-t) - U_{cl}\}\frac{\partial c}{\partial r} = \{U_{cl}x(1-t) - U_{ll}\}\frac{\partial n}{\partial r}$$

$$\Rightarrow \{U_{cc}x(1-t) - U_{cl}\}\frac{\partial c}{\partial r} = \{U_{cl}x(1-t) - U_{ll}\}\{\frac{\partial c}{\partial r} - 1\}\frac{1}{x(1-t)}$$

$$\Rightarrow \{U_{cc}x^2(1-t)^2 - U_{cl}x(1-t)\}\frac{\partial c}{\partial r} = \{U_{cl}x(1-t) - U_{ll}\}\frac{\partial c}{\partial r} - U_{cl}x(1-t) + U_{ll}$$

$$\Rightarrow \{U_{cc}x^2(1-t)^2 - 2U_{cl}x(1-t) + U_{ll}\}\frac{\partial c}{\partial r} = -U_{cl}x(1-t) + U_{ll}$$

$$\Rightarrow D\frac{\partial c}{\partial r} = -U_{cl}x(1-t) + U_{ll}$$

$$\Rightarrow \frac{\partial c}{\partial r} = \frac{U_{cl}x(1-t) - U_{ll}}{-D} > 0$$

Response of Pre-tax Income to Productivity

Pre-tax income is a function of the tax rate, transfer and the productivity level,

$$y(r,t,x) = xn(r,x(1-t))$$

For individuals with $x \leq x_0$ (i.e. not choose to work) $y = 0$ implying $\frac{\partial y}{\partial x} = 0$. For individuals for whom $n > 0$,

$$\frac{\partial y}{\partial x} = n + x\ \frac{\partial n}{\partial x}$$

Differentiating the original utility maximization problem with respect to x,

$$(1-t)\left\{\left[U_{cc}(1-t)\left(\frac{\partial n}{\partial x}x + n\right) - U_{cl}\frac{\partial n}{\partial x}\right]x + U_c\right\} = U_{cl}\left\{(1-t)\left(\frac{\partial n}{\partial x}x + n\right)\right\} - U_{ll}\frac{\partial n}{\partial x}$$

$$\Rightarrow (1-t)\left\{U_{cc}(1-t)\frac{\partial n}{\partial x}x + U_{cc}(1-t)n - U_{cl}\frac{\partial n}{\partial x}\right\}x + (1-t)U_c$$
$$= U_{cl}(1-t)\frac{\partial n}{\partial x}x + U_{cl}(1-t)n - U_{ll}\frac{\partial n}{\partial x}$$

$$\Longrightarrow \mathrm{U_{cc}}(1-\mathrm{t})^2\mathrm{x}^2\frac{\partial \mathrm{n}}{\partial \mathrm{x}} + \mathrm{U_{cc}}(1-\mathrm{t})^2\mathrm{nx} - \mathrm{U_{cl}}(1-\mathrm{t})\mathrm{x}\frac{\partial \mathrm{n}}{\partial \mathrm{x}} + (1-\mathrm{t})\mathrm{U_c}$$
$$= \mathrm{U_{cl}}(1-\mathrm{t})\frac{\partial \mathrm{n}}{\partial \mathrm{x}}\mathrm{x} + \mathrm{U_{cl}}(1-\mathrm{t})\mathrm{n} - \mathrm{U_{ll}}\frac{\partial \mathrm{n}}{\partial \mathrm{x}}$$

$$\Longrightarrow \frac{\partial \mathrm{n}}{\partial \mathrm{x}}\{\mathrm{U_{cc}}(1-\mathrm{t})^2\mathrm{x}^2 - 2\,\mathrm{U_{cl}}(1-\mathrm{t})\mathrm{x} + \mathrm{U_{ll}}\} = -\mathrm{U_{cc}}(1-\mathrm{t})^2\mathrm{nx} + \mathrm{U_{cl}}(1-\mathrm{t})\mathrm{n} - (1-\mathrm{t})\mathrm{U_c}$$

$$\Longrightarrow \frac{\partial \mathrm{n}}{\partial \mathrm{x}}\mathrm{D} = -\mathrm{U_{cc}}(1-\mathrm{t})^2\mathrm{nx} + \mathrm{U_{cl}}(1-\mathrm{t})\mathrm{n} - (1-\mathrm{t})\mathrm{U_c}$$

$$\Longrightarrow \frac{\partial \mathrm{n}}{\partial \mathrm{x}} = \frac{\mathrm{U_{cc}}(1-\mathrm{t})^2\mathrm{nx} - \mathrm{U_{cl}}(1-\mathrm{t})\mathrm{n} + (1-\mathrm{t})\mathrm{U_c}}{-\mathrm{D}}$$

Substituting this back into the partial derivative of earned income with respect to productivity,

$$\frac{\partial \mathrm{y}}{\partial \mathrm{x}} = \mathrm{n} + \mathrm{x}\,\frac{\mathrm{U_c}(1-\mathrm{t})\mathrm{x} + \mathrm{n}\{\mathrm{U_{cl}}(1-\mathrm{t})\mathrm{x} - \mathrm{U_{ll}}\}}{-\mathrm{D}} > 0$$

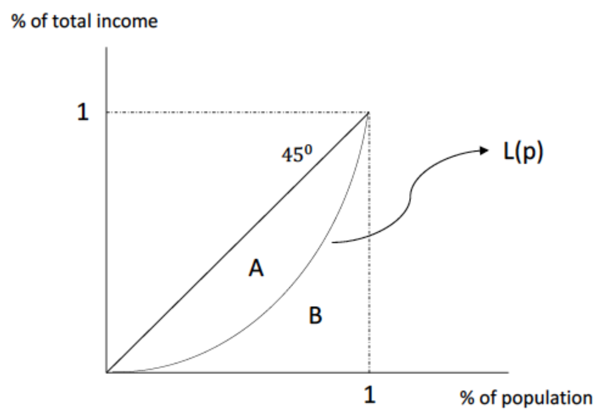
So, $\frac{\partial \mathrm{y}}{\partial \mathrm{x}}$ is positive for all $\mathrm{x} > \mathrm{x_0}$.

A.2. Technical Derivations of Lambert (2001).

Lorenz Curve, $L(p)$ represents the percentage of total income belonging to the poorest p % of the population and can be written as;

$$L(p) = \int_0^y \frac{xf(x)dx}{\mu} = \int_0^{F^{-1}(p)} \frac{xf(x)dx}{\mu}$$

where $p = F(y)$ is the rank of an individual across income distribution.



The Gini coefficient is given by;

$$G = \frac{A}{A+B} = \frac{A}{A + (\frac{1}{2} - A)} = \frac{A}{A + (\frac{1}{2} - A)} = 2A = 2\left(\frac{1}{2} - B\right) = 1 - 2B$$

Substituting in the Lorenz Curve;

$$G = 1 - 2\int_0^1 L(p)dp$$

$$= 1 - 2[\left(pL(p)\right)|_0^1 - \int_0^1 pL'(p)dp]$$

$$= 1 - 2[L(1)(1) - L(0)(0) - \int_0^1 pL'(p)dp]$$

$$= -1 + 2\int_0^1 pL'(p)dp$$

$$= -1 + 2\int_0^1 pL'(p)dp$$

$$= -1 + \frac{2}{\mu}\int_0^1 p\frac{1}{f(F^{-1}(p))}F^{-1}(p)f(F^{-1}(p))dp$$

$$= -1 + \frac{2}{\mu}\int_0^1 pF^{-1}(p)dp$$

$$= -1 + \frac{2}{\mu}\int_0^\infty F(y)yf(y)dy$$

since $p = F(y)$ and $dp = f(y)dy$.

The covariance of y and $F(y)$ is given by;

$$Cov[y, F(y)] = \int_0^\infty yF(y)f(y)dy - \mu\int_0^\infty F(y)f(y)dy$$

$$= \int_0^\infty yF(y)f(y)dy - \mu\frac{1}{2}[F(y)]^2|_0^\infty$$

$$= \int_0^\infty yF(y)f(y)dy - \frac{1}{2}\mu$$

$$\Rightarrow \int_0^\infty yF(y)f(y)dy = Cov[y, F(y)] + \frac{1}{2}\mu$$

$$\Longrightarrow \; G = -1 + \frac{2}{\mu}[\,Cov[y, F(y)] + \frac{1}{2}\mu]$$

$$\Longrightarrow \; G = \tfrac{2}{\mu} Cov[y, F(y)]$$