INTER-TEMPORAL SUSTAINABILITY OF FISCAL REDISTRIBUTION: A METHODOLOGICAL FRAMEWORK

José María Fanelli

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ABSTRACT

This chapter develops a methodological framework to study the linkages between fiscal redistributions, fiscal sustainability, and the government's wealth constraint. The framework includes demographic factors and income strata, shows the connections between the concepts used in the CEQ (Commitment to Equity Institute) and NTA (National Transfer Accounts) databases, and suggests possible synergies and directions for further data collection and research efforts. We conclude that more research is needed, first, about the role of public wealth including all assets in the government's balance sheet and, second, about the distributional consequences – on income as well as wealth – of policies regarding fiscal sustainability, intergenerational transfers that finance the demand for life-cycle wealth, and the management of publicly-owned natural resources. We also show that the framework is useful to connecting the two approaches to sustainability: the one concerning fiscal soundness and the one concerning development. With regard to policies, the implications indicate that sustainability tests should be part of the design of redistribution initiatives, that these initiatives must consider the demographic transition, and that fiscal redistributions may ultimately deplete the stock of natural resources without ensuring a compensatory accumulation of reproducible capital if they do not take adjusted government savings and capital gains into account.

JEL Codes: E62; J11.

Keywords: Fiscal policy, Demographics

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Inter-temporal Sustainability of Fiscal Redistribution

A Methodological Framework

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September, 2018

Abstract

This chapter develops a methodological framework to study the linkages between fiscal redistributions, fiscal sustainability, and the government's wealth constraint. The framework includes demographic factors and income strata, shows the connections between the concepts used in the CEQ (Commitment to Equity Institute) and NTA (National Transfer Accounts) databases, and suggests possible synergies and directions for further data collection and research efforts. We conclude that more research is needed, first, about the role of public wealth including all assets in the government's balance sheet and, second, about the distributional consequences – on income as well as wealth – of policies regarding fiscal sustainability, intergenerational transfers that finance the demand for life-cycle wealth, and the management of publicly-owned natural resources. We also show that the framework is useful to connecting the two approaches to sustainability: the one concerning fiscal soundness and the one concerning development. With regard to policies, the implications indicate that sustainability tests should be part of the design of redistribution initiatives, that these initiatives must consider the demographic transition, and that fiscal redistributions may ultimately deplete the stock of natural resources without ensuring a compensatory accumulation of reproducible capital if they do not take adjusted government savings and capital gains into account.

JEL Codes: Fiscal Policy E62; Demographic Economics J11
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Executive Summary

This chapter develops a methodological framework to study the linkages between fiscal redistributions, fiscal sustainability, and the government's wealth constraint. The framework includes demographic factors and income strata, shows the connections between the concepts used in the CEQ (Commitment to Equity Institute) and NTA (National Transfer Accounts) databases, and suggests possible synergies and directions for further data collection and research efforts.

We conclude that more research is needed, first, about the role of public wealth including all assets in the government's balance sheet and, second, about the distributional consequences – on income as well as wealth – of policies regarding fiscal sustainability, intergenerational transfers that finance the demand for life-cycle wealth, and the management of publicly-owned natural resources. We also show that the framework is useful to connecting the two approaches to sustainability: the one concerning fiscal soundness and the one concerning development.

Concerning policies, the following implications deserve mention.

(1) Fiscal sustainability tests should be part of any significant initiative involving fiscal redistributions. Policies that do not pass the sustainability tests could undermine the ability of the state to improve income distribution, protect the poor and create a growth friendly environment over time, giving rise to socially disruptive phenomena, such as "lost decades".

(2) In the case of natural resource-rich countries it is particularly relevant to consider that they may ultimately deplete the stock of natural capital without ensuring a compensatory accumulation of reproducible capital if redistribution initiatives do not evaluate carefully the consequences for adjusted savings, taking into account capital gains.

(3) In the context of the ongoing demographic transition, even if the parameters of fiscal redistributions are maintained, the changes in the weight of the different cohorts in the total population will modify the size of fiscal redistributions. This is one important reason why demography must be taken into consideration when designing fiscal redistributions and assessing sustainability. It also matters to income and wealth distribution to the extent that income distribution differs among cohorts.

(4) Transfers associated with the social security system are a substantial part of public redistributions and a key determinant of both the life-cycle deficit and the government deficit. The ways in which life-cycle deficits and the demand for the life-cycle wealth of each cohort are financed impinge significantly on the distribution of wealth between the public and the private sectors and across-generations as well.

(5) Whether we consider pensions as deferred income or not matters for the distribution of wealth between the public and private sectors and, probably, for public opinion's perception of the significance of public redistributions. Public opinion's misperception can easily result in a demand for life-cycle wealth that cannot be satisfied, given the economy's capacity to accumulate wealth and the restrictions imposed by sustainable development on the trajectory of natural capital.

Introduction[1]

Fiscal redistributions can have important consequences for both the allocation of wealth – including natural capital – across generations and fiscal sustainability.[2] In turn, when fiscal sustainability is under scrutiny, the ability of the state to improve income distribution and protect the poor might be affected for long periods. The following points will help to show the relevance of the issue.

First, taxes and transfers that seek to bring about changes in income distribution typically modify the inter-temporal allocation of fiscal revenues, expenses, and the primary balance, implying that fiscal sustainability might be at stake and, hence, could limit the public sector's ability to access credit markets. This suggests that sustainability tests should be part and parcel of the design of redistribution policies in order to check for inter-temporal stability and reduce the probability of disorderly fiscal adjustments.

Second, the existing structure of fiscal redistributions or changes in it must be financed and some financial strategies may have undesirable consequences for future generations. If the redistribution is financed with debt to avoid increasing prevailing taxes, the financial burden will be shifted toward future generations and the way in which the shift impacts on each of the future generations will not be independent of the stage of the demographic transition that the economy is experiencing. In this regard, policy makers should take into account that children and unborn generations cannot participate in the markets and may have a weak or no voice in the political arena.

Third, fiscal redistributions may affect natural capital. In particular, if redistribution initiatives are financed with rents from natural resources, they may ultimately deplete natural capital leaving no accumulation of reproducible capital to compensate for such depletion as required, for example, by the criterion of weak sustainability (Hartwick, 1977; see also Hamilton, 2008). In addition, when positive but transitory shocks occur, such as improvements in the terms of trade in natural resource-rich economies, the short-run political economy equilibrium may result in fiscal redistributions that are progressive and favor the poor but cannot be maintained under normal circumstances once the positive shock has passed. To avoid reducing progressive expenditures when rents are falling, state-owned firms frequently increase the level of oil and other natural resources extraction beyond the optimum. A closely related problem has to do with non-targeted subsidies embodied in the prices of energy in oil rich countries, which may not only lead to regressive results but also to negative effects on the stock of natural capital when lower prices provide stronger incentives for the excessive consumption of energy.[3] In all these cases fiscal redistributions would be financed by depleting future generations' natural resources and, under such circumstances, fiscal sustainability might appear to be ensured when,

[1] This article has benefited from comments by an anonymous reviewer, Nora Lustig, Ramiro Albrieu and the participants in the CEQ Handbook 2020 Workshop, November, 2017 and the LACEA CEQ Panel, November, 2017

[2] In this paper we call "fiscal redistributions" the difference between households' final income and households' market income that results from the incidence of taxes, expenditures, and transfers that are primarily intended to produce changes in income distribution. We utilize the definitions corresponding to the CEQ (Commitment to Equity Institute) methodology, see section 1 and Lustig and Higgins (2017).

[3] See for example De la Torre et al. (2009) and Fanelli et al. (2015).

in fact, it would not be. These factors are highly relevant in low- and middle-income countries where natural capital represents a much higher proportion of total wealth (World Bank, 2011) and, consequently, it is highly probable that the state finances public policies based on natural resources rents.

The main purpose of this chapter is to present a set of methodological tools to address these types of problems. We focus on the linkages between fiscal redistributions, fiscal sustainability and the government's wealth constraint. To this end, we will use concepts developed in four literatures: fiscal incidence (Lustig N. and S. Higgins, 2017), fiscal sustainability (Escolano, 2010), sustainable development (Dasgupta, 2009, Neumeyer, 2010, United Nations, 2015), and the demographic transition (Mason and Lee, 2011). Two additional objectives are to identify new research questions and new data requirements.

The structure of the chapter is as follows. The second section, after this introduction, elaborates on three central concepts of our analysis – fiscal redistributions, public wealth, and fiscal sustainability – and the linkages between them. We use the set of income concepts developed by the Commitment to Equity Institute (CEQ)[4] to define the components of fiscal redistributions and, then, present the concept of public wealth and show what the fiscal sustainability conditions are for a given set of fiscal redistributions. The section addresses two additional issues: the connections between fiscal sustainability and public wealth; and the relation between this latter wealth, natural resources and (weak) development sustainability, when the government owns the natural resources. We also examine the consequences on the distribution of wealth between the public and the private sectors when capital gains are considered, in line with Vincent et al. (1997). The third section addresses the demographic dimension. We first present a set of concepts developed by the National Transfer Accounts project (NTA) to conceptualize and measure the economic consequences of the demographic transition.[5] Based on such concepts, we study the cross-cohort distribution of income and wealth, on the one hand, and the relationship between sustainability and fiscal redistributions on the other. The fourth section introduces disaggregation by income strata and investigates the relations with cohorts and aggregate wealth. This is necessary to study the consequences of changes in taxes or transfers whose primary purpose is to ensure fiscal sustainability. The concluding section comments on a set of policy implications that follow from our methodological framework. The chapter has two Annexes: Annex 1 modifies the framework to analyze the consequences of assuming that the contributions to social security are forced savings rather than a tax and, hence, the associated transfers constitute the perception of deferred income. Annex 2 presents a list of the framework's variables.

[4] See Lustig and Higgins (2017).

[5] On the National Transfers Accounts methodology see Mason and Lee (2011).

2. Income Concepts, Fiscal redistributions, and Sustainability

In this section, first, we present the income concepts utilized by CEQ Institute researchers – that is, market income, disposable income, consumable income, and final income. Second, we define the government's intertemporal budget constraint in terms of such concepts and include natural resources in the government's balance sheet. Third we discuss the relation of our approach to the notion of fiscal sustainability commonly used in policy making analyses, for example, in the case of the IMF's sustainability exercises (Escolano, 2010) and identify assumptions that are frequently made concerning government wealth constraints. Finally, we analyze the linkages between rents from natural resources, wealth distribution and fiscal redistributions.

CEQ Income Concepts

We begin by defining *market income* (Y_t^M) as the sum of market labor income (Y_t^L) and the market income stemming from accumulated assets (Y_t^A) before taxes. Income from assets includes private transfers such as private pensions and remittances. Hence, market income can be written as:

(1) $Y_t^M = Y_t^L + Y_t^A$

In addition to market income, *disposable income* (Y_t^D) takes into account direct cash and near cash transfers – the sum of pension transfers (G_t^A) and other transfers (G_t^O) – net of employee contributions to social security (T_t^A) and personal taxes (T_t^Y). Examples of other transfers are conditional and unconditional cash transfers, school feeding programs, and free food transfers. Hence:

(2) $Y_t^D = Y_t^L + Y_t^A + G_t^O + G_t^A - T_t^A - T_t^Y$

Consumable income (Y_t^C) is obtained by adding indirect subsidies (G_t^I) to energy, food and other general targeted subsidies and subtracting indirect taxes (T_t^I) from disposable income. So, consumable income (Y_t^C) is:

(3) $Y_t^C = Y_t^L + Y_t^A + G_t^O + G_t^A + G_t^I - T_t^A - T_t^Y - T_t^I$

Final income (Y_t^F) is calculated by adding expenditures in kind related to education (G_t^E) and health (G_t^H) and subtracting fees (T_t^F) from the previous income concepts

(4) $Y_t^F = Y_t^L + Y_t^A + G_t^O + G_t^A + G_t^I + G_t^E + G_t^H - T_t^A - T_t^Y - T_t^I - T_t^F$

Based on these income concepts, we define (net) *fiscal redistributions* (N_t^D) as the difference between market income and final income, that is:

(5) $N_t^D = Y_t^F - Y_t^M$

The variable N_t^D can be interpreted as the net overall costs that the public sector must incur to implement a specific set of fiscal redistributions aimed at achieving a given target concerning income redistribution. This variable connects two central aspects of fiscal policies:

redistributions and sustainability. To examine specific issues concerning the effects of public policies on income distribution, fiscal redistributions can be defined more narrowly. More specifically, N_t^D can be defined in two alternative ways: as a difference between market income and disposable income, or as the difference between market income and consumable income. However, in the case of our analysis, it is the variable N_t^D that will play the central role in showing the linkages between redistributive initiatives, fiscal sustainability and demography. This is so because it is the ampler definition of fiscal redistributions and, so, it is more suitable for examining the consequences at the macroeconomic level. But, in any case, the methodological framework that we will develop can be easily adapted to any of the above definitions of fiscal redistributions.

Our next step will be to define the fiscal deficit in terms of N_t^D. The primary fiscal deficit (D_t^{PG}) is the government net borrowing, excluding interest payments on consolidated government liabilities, which equals the difference between primary expenditures and taxes and other revenues. In addition to the items that we have already presented, primary expenditures include government investment (I_t^G) and a variety of other items associated with the provision of public goods, which we will call G_t^R. Taxes, in turn, usually comprise a number of miscellaneous revenues (including corporate taxes) besides the set of taxes that we have introduced above; we will call them T_t^R. If the capital accumulated on the basis of public investment generates an income (for example, highway tolls or hospital fees) it is included in T_t^R. In other parts of this chapter, when necessary to discuss specific problems, we will change the assumptions concerning the returns of public investment. We also introduce a variable that stands for the net incidence of the miscellaneous components of the budget that are not part of what we have called fiscal redistributions:

(6) $N_t^R = G_t^R - T_t^R.$

The primary fiscal deficit can then be expressed as:

(7) $D_t^{PG} = N_t^D + N_t^R + I_t^G - E_t^G$

where E_t^G are rents from natural resources, which can take the form of dividends from government-owned natural resource firms or royalties and may account for a significant share of fiscal revenues in natural-resource rich countries. E_t^G is equal to the variation in the quantity of natural resources ($\Delta Q_t^G < 0$) times the value of the rents of natural resources p_t (the price net of marginal costs)[6]. Hence, $E_t^G = -p_t \Delta Q_t^G$

Public wealth constraint

The assets that make up public wealth are reproducible capital (K_t^G) and non-renewable natural resources (Q_t^G). If B_t^G is the stock of government debt net of financial assets held by the government, the government's net worth, W_t^G, can be defined as:

(8) $W_t^G = K_t^G + p_t Q_t^G - B_t^G$

[6] Of course, ΔQ_t^G can be positive as a consequence of discoveries, but we simplify by not including discoveries.

When the stock of natural resources is included in the government's balance sheet, a number of particularities have to be considered. The two most relevant to our analysis are the definition of net or "adjusted" savings to take into account the depletion of natural resources and the capital gains originating in changes in the value of rents.

To calculate net or adjusted savings (S_t^G) we have to deduct both the depreciation of capital ($\varsigma\, K_t^G$) and resource depletion ($\bar{E}_t^G$) from gross savings. Hence, if *r* is the interest rate – which we simplify by assuming constant – we can write:

(9) $S_t^G = E_t^G + rK_{t-1}^G - rB_{t-1}^G - N_t^D - N_t^R - \varsigma\, K_{t-1}^G - \bar{E}_t^G$

Gross savings, in turn, provide the funds to finance the accumulation of capital and to repay public debt:

(10) $S_t^G + \bar{E}_t^G + \varsigma\, K_{t-1}^G = I_t^G - \Delta B_t^G$

These definitions of gross and net savings are consistent with the sustainable approach to development (see Hamilton, 2008). However, this definition of adjusted savings is less restrictive than that of the World Bank, which excludes additional items from gross savings (see World Bank, 2011). We will ignore those items, as well as the capital depreciation term ($\varsigma\, K_t^G$), because they play no particular role in our analysis – and can easily be included if necessary. We focus on the way in which rent revenues and the depletion of non-renewable resources can influence fiscal redistribution policies. Note, nonetheless, that different approaches exist concerning the adjustment of savings to account for depletion (see Neumayer, 2010). To reflect this fact, we define: $\bar{E}_t^G = -m_t p_t \Delta Q_t^G$, where: 0≤ m_t ≤1; $\forall t$. In the case of the Hartwick (1977) rule, $m_t = 1$ and $S_t^G \geq 0$, which implies that fiscal redistributions are subject to the restriction: $N_t^D \leq r(K_{t-1}^G - B_{t-1}^G) - N_t^R$. If, instead, we followed the El Serafy (1989) approach: 0<m_t<1. In the usual National Accounts' calculations $m_t = 0$. In the two latter cases it is easier to comply with $S_t^G \geq 0$ and, consequently, there is more room to expand fiscal redistributions: $N_t^D \leq (1 - m_t)E_t^G + r(K_{t-1}^G - B_{t-1}^G) - N_t^R$. In what follows, we assume $m_t = 1$ in line with the World Bank's measurement of adjusted savings.

The increase in the value of the portion of wealth held in natural resources can be decomposed as follows:

(11) $p_t Q_t^G - p_{t-1} Q_{t-1}^G = \Delta p_t Q_{t-1}^G + p_t \Delta Q_t^G = \hat{p}_t p_{t-1} Q_{t-1}^G + p_t \Delta Q_t^G$

Capital gains stem from changes in the value of rents. In each period, capital gains amount to: $\Delta p_t Q_{t-1}^G = \hat{p}_t p_{t-1} Q_{t-1}^G$. If these gains are different from zero, the increase in wealth (ΔW_t^G) differs from savings. Since natural capital is not usually, or only partially, recorded in public sector balance sheets, capital gains associated with the stock of natural resources are mostly ignored when stating fiscal sustainability conditions. When capital gains are considered, government wealth (W_t^G)evolves according to:

(12) $W_t^G = W_{t-1}^G + S_t^G + \hat{p}_t p_{t-1} Q_{t-1}^G = K_{t-1}^G - B_{t-1}^G + I_t^G - \Delta B_t^G + p_t Q_t^G$

Where $I_t^G = \Delta K_t^G$ stands for government investment. If, additionally, we assume – in line with the Hotelling rule (Hotelling, 1931) – that $r = \hat{p}_t$, capital gains can be expressed as: $r p_{t-1} Q_{t-1}^G$.

Whenever $r \neq \hat{p}_t$ as a consequence of a shock, the activity of speculators will induce a rapid "jump" in stock prices so as to restore the parity, giving rise to a once-and-for-all variation in the value of under-the-ground resources and, consequently, of public wealth. So, if natural resources were owned by the government, it would be the treasury and not the private sector that would be favored by capital gains. In fact, in a closed economy where the public sector owns a natural resource that is used as an input for production or for consumption – such as oil – when $r = \hat{p}_t > 0$, the private sector will become relatively less rich than the public sector because of capital gains. On the other hand, if the country were a net oil exporter, part of the capital gains would be at the cost of the rest of the world. These gains represent the capitalized value of the increase in national income induced by the increase in p_t (see Vincent et al., 1997).

In addition, in countries where natural resources account for a relevant share of exports, when sizable unexpected positive terms-of-trade shocks occur (p_t jumps), the fiscal space typically widens substantially because of the increase in the value of the flow of rents E_t^G. Fiscal sustainability might also improve for two reasons. First, since the value of domestic assets that can be used as collateral is higher, the public debt ratio that market participants perceive as sustainable might increase. Second, the improvement in the agents' perception of the treasury's ability to pay might reduce the interest rate and thus, as we will see, have a direct positive impact on fiscal sustainability. Under these circumstances, as we mentioned above, the political pressures on the government to implement bolder fiscal redistributions will be typically stronger and the consequences of mistaking a transitory shock for a permanent one can be very damaging to the stability of fiscal redistributions and, hence, fiscal sustainability. The consequences of these simultaneous distributional and financial changes on the macroeconomic and political economy dimensions can give rise to symptoms of the natural resource course

Under many circumstances – especially when studying the demographic dimension – we conduct the analysis in per capita terms. Therefore, to simplify the notation, we will use lowercase letters to express the value of the variables in per capita terms. Hence, for example, per capita income is: $y_t = \frac{Y_t}{X_t}$, where X_t stands for the total population. In addition, we will use Greek letters when we express a per capita variable as a ratio of per capita income. Therefore, fiscal redistributions, for instance, will be: $\eta_t^D = \frac{n_t^D}{y_t}$. Note, however, that in the case of financial variables, we will use a tilde instead of a Greek letter to express the per capita variable as a ratio of per capita income. In this way, $\tilde{b}_t^G = \frac{b_t^G}{y_t}$ is the stock of net government debt per capita as a ratio of per capita income. Using these conventions, we can express (12) as a ratio of GDP in the following way:

$$(13)\ \omega_t^G = \frac{\omega_{t-1}^G}{1+g} + \sigma_t^G + \hat{p}_t p_{t-1} \frac{\xi_{t-1}^G}{1+g}$$

where: $\omega_t^G = \frac{W_t^G}{Y_t}$; $\sigma_t^G = \frac{S_t^G}{Y_t}$; $\xi_{t-1}^G = \frac{Q_t^G}{Y_t}$; $\hat{p}_t = \frac{\Delta p_t}{p_{t-1}}$; and g is the GDP growth rate. Since we have assumed that $E_t^G = \bar{E}_t^G$, the expression for the public wealth can also be written as:

$$\omega_t^G = (1+\lambda)\left(\kappa_{t-1}^G - \tilde{b}_{t-1}^G\right) - (\eta_t^D + \eta_t^R) + (1+\lambda_t^*)p_{t-1}\xi_0^G$$

where we defined: $\kappa_t^G = \frac{K_t^G}{Y_t}$; $\eta_t^R = \frac{N_t^R}{Y_t}$; $1 + \lambda = \frac{1+r}{1+g}$ and $1 + \lambda_1^* = \frac{1+\hat{p}_t}{1+g}$. Under the Hotelling rule:$1 + \lambda_t^* = 1 + \lambda$ and, therefore, in period *N*, the government's net worth as a ratio of GDP will be:

(14) $\omega_N^G = \kappa_N^G - \tilde{b}_N^G + p_N \xi_N^G = (1+\lambda)^N \left(\kappa_0^G + p_0 \xi_0^G - \tilde{b}_0^G\right) - \sum_{t=1}^{N} (1+\lambda)^{N-t} \left(\eta_t^D + \eta_t^R\right)$.

Expressing (14) at present value we obtain:

(15) $\omega_N^G (1+\lambda)^{-N} = \left(\kappa_0^G + p_0 \xi_0^G - \tilde{b}_0^G\right) - \sum_{t=1}^{N} (1+\lambda)^{-t} \left(\eta_t^D + \eta_t^R\right)$

Assuming that the No-Ponzi Game condition holds, that *r>g,* and taking into account that non-renewable resources, by definition, have a finite duration, if we let $N \rightarrow \infty$, it follows that the intertemporal budget constraint that fiscal redistributions have to abide by is:

(16) $\sum_{t=1}^{\infty} (1+\lambda)^{-t} \eta_t^D = \left(\kappa_0^G - \tilde{b}_0^G p_0 + \xi_0^G\right) - \sum_{t=1}^{\infty} (1+\lambda)^{-t} \eta_t^R$

Our definition of adjusted public savings, with $\bar{\varepsilon}_t^G = \varepsilon_t^G$, ensures that the full amount of rents received will be saved and used to either accumulate capital or reduce public debt. If, instead, the authorities followed the El Serafy approach and only part of ε_t^G were allocated to finance depletion, there would be more fiscal space available to finance η_t^D. In the remainder of the chapter we will use the wealth constraint (13) as our frame of reference and make different simplifying assumptions to either focus on specific issues or adapt to the approach that is customarily applied in economic policy making. We will now show the relationship between (13) and the usual approach to fiscal sustainability.

Fiscal Sustainability

Policy makers assessing fiscal sustainability emphasize debt sustainability. Neither reproducible capital nor the stocks of natural resources and their depletion are fully taken into consideration (see Escolano, 2010). The budget constraint, however, does consider the rents that the government receives from natural resources in the form of dividends from public firms or royalty payments. In turn, public investment is an expenditure with no counterpart in the accumulation of capital in the government balance sheet. The investment in financial assets, nonetheless, is taken into account to the extent that the stock of debt is net of the financial assets that the government holds. Therefore, we can express the recursive equation governing the dynamics of the public debt to income ratio as:[7]

(17) $\tilde{b}_t^G = (1+\lambda)\tilde{b}_{t-1}^G + \eta_t^D + \eta_t^R + \iota_t^G - \varepsilon_t^g$

Liquidity considerations are probably an important reason to exclude capital gains associated with future dividends or royalty payments from equation (17), while including rents received, $\varepsilon_t^G = \frac{E_t^G}{Y_t}$. Capital markets are far from perfect and, consequently, capitalized gains might be very difficult to realize over a short period. Liquidity may also be one of the reasons why $\bar{\varepsilon}_t^G = \frac{\bar{E}_t^G}{Y_t}$ is

[7] This follows from:$\frac{B_t}{Y_t} = \frac{(1+r)\,B_{t-1}}{Y_{t-1}}\frac{Y_{t-1}}{Y_t} + \frac{N_t^D}{Y_t} + \frac{N_t^R}{Y_t} + \frac{I_t^G}{Y_t} - \frac{E_t^G}{Y_t} ==> \tilde{b}_t = \frac{1+r}{1+g}\tilde{b}_{t-1} + \eta_t^D + \eta_t^R + \iota_t^G - \varepsilon_t^G$.

not included in the budget, missing the opportunity to make policy decisions based on adjusted rather than gross government savings. When access to credit markets becomes difficult, rents are a source of liquidity and will be available to the extent that they are not invested in reproducible capital to compensate for depletion. In addition, investment in reproducible capital ($\iota_t = \frac{I_t^G}{X_t}$) may not be politically palatable to the extent that they represent an increment in public expenditures. In short, liquidity squeezes and capital market imperfections undoubtedly hinder the policy maker's ability to strike an appropriate balance between efficiency, intra- and intergenerational equity, and fiscal sustainability

The solution of the difference equation (17) is:

(18) $\tilde{b}_N = (1+\lambda)^N \, \tilde{b}_0 + \sum_{t=1}^{N}(1+\lambda)^{N-t}\,(\eta_t^D + \eta_t^R + \iota_t^G - \varepsilon_t^G)$,

This implies that policies that contribute to determining the allocation of resources between η_t^D, η_t^R, and ι_t^G over time, as well as the rents from state-owned assets ε_t^G, will have a bearing on the trajectory of the debt/per capita income ratio. It also shows, as is well known, that the evolution of the interest rate-growth differential (λ , which we will call the "effective" interest rate) has an effect on the path of public sector liabilities[8]. We can write the previous equation in present-value terms as:

(19) $(1+\lambda)^{-N}\tilde{b}_N = \tilde{b}_0 + \sum_{t=1}^{N}(1+\lambda)^{-t}\,(\eta_t^D + \eta_t^R + \iota_t^G - \varepsilon_t^G)$

This is the inter-temporal version of the government budget constraint in the "debt sustainability" approach. Consequently, for the level of public indebtedness to be sustainable, it is necessary to impose the no-Ponzi-game condition, which means that the government cannot service the interest and principal on its debt on a regular basis. This implies that the fiscal authorities must respect the following constraint:

(20) $\lim_{N\to\infty}(1+\lambda)^{-N}\,\tilde{b}_N = 0$

The government budget constraint then becomes:

(21) $\tilde{b}_0 = \sum_{t=1}^{\infty}(1+\lambda)^{-t}\,(\varepsilon_t^G - \eta_t^D - \eta_t^R - \iota_t^G) = -\sum_{t=1}^{\infty}(1+\lambda)^{-t}\,\tilde{d}_t^{PG}$

Where $\tilde{d}_t^{PG} = \frac{D_t^{PG}}{Y_t}$. This means that the surpluses that the government plans to run in the future must be equal to the value of the current stock of debt and, consequently, the inter-temporal restriction that the sequence of fiscal redistributions must respect over time will be:

(22) $\sum_{t=1}^{\infty}(1+\lambda)^{-t}\,\eta_t^D = \sum_{t=1}^{\infty}(1+\lambda)^{-t}\,(\varepsilon_t^G - \eta_t^R - \iota_t^G) - \tilde{b}_0$

Since this expression ignores some government-owned assets, it differs from (16), which does include $\kappa_0^G + \xi_0^G$. The restriction on the fiscal redistributions would be softer if the stocks of natural resources and natural capital were taking into account in (22). For the sake of simplicity, we are omitting the role of human capital although this kind of capital would play a role similar

[8] If λ is not constant and equal to λ_t at time t, the solution is:
$\tilde{b}_N = \tilde{b}_0 \prod_{t=1}^{N}(1+\lambda_t) + \sum_{t=1}^{N}\prod_{i=t+1}^{N}(1+\lambda_i)\,(\eta_t^D + \eta_t^R + \iota_t^G - \varepsilon_t^G)$,

to that of physical capital: as in the case of physical capital, equation (22) includes expenditures on education and health in calculating the primary deficit but excludes the accumulation of human capital as a source of social benefits. In addition, (22) implicitly assumes $\bar{\varepsilon}_t^G = 0$ and, consequently, restriction (22) does not exclude the possibility of fiscal sustainability being achieved at the cost of sacrificing the (weak) sustainability of the development process.

If, in order to meet constraint (22), the government were to implement a fiscal rule to maintain the ratio between the primary deficit and overall income constant, such a primary deficit would have to be: $\tilde{d}^{PG^*} = -\lambda \tilde{b}_{0,}$ because $\sum_{t=1}^{\infty}(1+\lambda)^{-t} = \frac{1}{\lambda}$. This implies that the government should run a surplus if it were a net debtor. Under these conditions, at each point in time, fiscal redistributions would face the restriction:

(23) $\eta_t^D = \varepsilon_t^G - \eta_t^R - \iota_t^G - \lambda\, \tilde{b}_0$

At each point in time then, fiscal redistributions (η_t^D) would compete with other items in the budget (η_t^R and ι_t^G). If the economy grew faster, the trade-off would be softer because the effective interest rate λ would be lower and the opposite would happen if there were an increase in the interest rate. This is why the "lost decades" situations are so disruptive to fiscal redistribution policies: they combine high interest rates – because of the increment in risk aversion – and low growth for long periods, making sustainability harder to achieve and constraining the government's ability to implement fiscal redistributions that aims to improve equity. Obviously, an improvement in the terms of trade that elevated ε_t^G via state-owned firms' profits would increase the fiscal space, making the implementation of fiscal redistributions easier. However, if the shock is transitory and the fiscal redistribution permanent, an inconsistency could arise once the shock disappears because the sustainability restriction must be respected throughout all periods. If the level of fiscal redistribution is maintained, the natural resources could be exhausted.

For political economy reasons and market failures the planning horizon is, in practice, short of infinite and fiscal rules that set a maximum public debt/per capita income value are, instead, frequent. If the fiscal authority sets $\tilde{b}_t = \tilde{b}^*$ as a sustainability rule, it follows that: $\lambda\tilde{b}^* = \varepsilon_t^G - \eta_t^D - \eta_t^R - \iota_t^G = \tilde{d}^{PG^*}$.

And the constraint on the costs of fiscal redistributions that holds at each time becomes:

(24) $\eta_t^{D^*} = \varepsilon_t^G - \eta_t^R - \iota_t^G - \lambda\tilde{b}^*$

If $\eta_t^D \neq \eta_t^{D^*}$, we can call ($\eta_t^D - \eta_t^{D^*}$) "fiscal sustainability gap". It represents the fiscal effort that would be necessary to meet the sustainability constraint expressed in terms of existing fiscal redistributions.

Two clarifications are in order. First, if the cause of the gap is that $\tilde{b}^* < \tilde{b}_t$ the treasury will have to make an additional effort to follow the rule because the surplus will probably have to be higher than $\lambda\tilde{b}^*$ for a number of periods until the stock of public debt achieves the target $\tilde{b}^*$. Once this target is achieved, the public debt/income ratio can be maintained on the basis of a primary surplus equal to λ$\tilde{b}^*$. Second, the rationale for a fiscal rule that set a constant primary surplus equal to λ $\tilde{b}^*$ has primarily to do with political economy and financial factors because, strictly speaking, the rule will maintain $\tilde{b}^* = \tilde{b}_0$ in the long run only if $\tilde{b}_t$ is already equal to $\tilde{b}^*$. The debt-stabilizing rule, in fact, should be set in terms of the overall deficit, $\tilde{d}^G$. The rule that makes $\tilde{b}_t$ asymptotically converge to $\tilde{b}^*$ is: $\tilde{d}^{G^*} = -\frac{g^n}{1+g^n}\tilde{b}^*$ where g^n is the nominal growth rate of income (see Escolano, 2010). With this caveat in mind, we will discuss the linkages between fiscal sustainability and redistributions in terms of the primary deficit because such deficit shows the stock and flow constraints in a clearer way and because the reference to the primary deficit is the norm rather the exception in policy making analysis. This makes sense because, concerning the market sentiment and political economy constraints, what usually matter the most in the short to medium run is the stabilization of the public debt ratio at a "reasonable" level.

Natural Resource Rents, Wealth, and Fiscal Redistributions

Fiscal sustainability restrictions give rise to complex issues in resource-rich countries and, consequently, fiscal redistributions should be carefully designed. But the issue is also relevant to resource-poor economies because of the effects of changes on international prices of imported resources, which reduce national income and could impinge on fiscal redistributions. To clarify this point, we will now further explore the relationship between natural resources, fiscal redistributions, and public wealth. We focus on two factors: the pattern of depletion over time and the effects of capital gains associated with changes in the value of scarcity rents. We assume that the public sector owns all natural resources.

We will use the expressions for the stocks of capital net of public debt (25) and natural resources (26) corresponding to period *N* to organize the analysis. These expressions are respectively:

(25) $\kappa_N^G - \tilde{b}_N^G = (1+\lambda)^N\left(\kappa_0^G - \tilde{b}_0^G\right) + \sum_{t=1}^N (1+\lambda)^{N-t}\varepsilon_t^G - \sum_{t=1}^N (1+\lambda)^{N-t}(\eta_t^D + \eta_t^R)$

(26) $p_N\xi_N^G = (1+\lambda)^N p_0\xi_0^G - \sum_{t=1}^N (1+\lambda)^{N-t}\varepsilon_t^G$

To highlight the problems facing fiscal authorities in a resource-rich developing country, we assume that r and p_t are exogenously determined by international markets and that the economy is a net exporter of renewable resources. We also make the simplifying assumption that the Hotelling rule holds and continue to assume $\varepsilon_t^G = \bar{\varepsilon}_t^G$.

Let us begin with the depletion pattern. Up to period *N*, the total amount of rents received will be: $\sum_{t=0}^N (1+\lambda)^{-t}\varepsilon_t^G$. If natural resources are depleted in *N* periods ($p_N\xi_N^G = 0$), from (26) it follows that the total amount of rents received will be equal to the stock of resources at the beginning of the period: $p_0\xi_0^G = \sum_{t=1}^N (1+\lambda)^{-t}\varepsilon_t^G$. Since we have imposed the condition that $\varepsilon_t^G = \bar{\varepsilon}_t^G$, the term $\sum_{t=0}^N (1+\lambda)^{-t}\varepsilon_t^G$ appears in equations (25) and (26), but with opposite signs. The term appears twice because capital accumulation fully offsets the depletion of natural resources over time. If we add (25) and (26) we obtain (14), that is, total wealth at the end of the period. Consequently, the distribution of rents and depletion over time is irrelevant to the value of $\kappa_N^G - \tilde{b}_N^G + p_N\xi_N^G$, the stock of wealth at period *N*.

In the real world, where market imperfections and political economy matter, the ΔQ_t^G sequence will not be optimally determined on the basis of a dynamic optimization model or, less ambitiously, trying to maintain the value of wealth for future generations.[9] To begin with, as we have mentioned, the fiscal authority does not often take into consideration the depletion of natural resources and adjusted savings may become negative (World Bank, 2011). Hence, in the real world, depletion policy matters, particularly for intergenerational equity. If the government sets a low N and, consequently, sets high absolute values for the $\Delta Q_t^G < 0$ sequence, fewer generations will benefit from rents. For example, the government might easily finance both η_t^D and i_t^D when $t \leq N$ but, afterwards, the Treasury might face a strong tradeoff between fiscal redistributions and capital accumulation.

In order to avoid the need for marked fiscal adjustments after period N – and seek intergenerational equity – ensuring that $\bar{\varepsilon}_t^G - \varepsilon_t^G = 0$ and $\sigma_t^G \geq 0$ appears to be a sensible strategy. However, this may not be the case. If public savings are positive but the accumulation of reproducible capital takes the form of, say, investment in infrastructure – or education – with no or partial user charge, the Treasury might not recover the funds invested. And, if fiscal sustainability were in jeopardy, the fiscal authority would have to implement undesired changes in fiscal redistributions. The government might, of course, utilize rents to repay debt instead of investing them and, eventually, $\tilde{b}_t^G$ might even become negative. This would be the case of a country that accumulated a sovereign wealth fund.[10] This policy would reinforce sustainability but it could be at the cost of weakening capital accumulation, which could have a higher rate of social return[11]. Furthermore, when resources remain under the ground it is as if the fiscal authority were systematically reinvesting the capital gains and, consequently, natural resource reserves grow at the rate r. If the resources are extracted and converted into productive capital, on the other hand, the rents, ε_t^G, may or may not be reinvested in productive capital and the same is true of the future proceeds ($r\varepsilon_t^G$). Therefore, because of political economy constraints, the government may decide not to extract the resource to prevent it from being squandered.

The expected and unexpected changes in the value of scarcity rents give rise to capital gains that may be partially or totally overlooked when planning fiscal redistributions. To illustrate the point, let us assume the limiting case of no depletion ($\varepsilon_t^G = 0, \forall t$). The value of the stock of resources (ξ_t^G) will increase at the effective rate λ. In period N, the value of natural resource wealth will be $p_N \xi_N^G = (1+\lambda)^N p_0 \xi_0^G$. However, if the resources are not appropriately recorded in the government's balance sheet, the increase in government wealth will not always be correctly considered.

Capital gains are not neutral for wealth and income distribution because those who buy the resources – domestic consumers or the rest of the world – face systematically increasing prices. At the domestic level, if natural resources are owned by the government, public wealth increases compared to private wealth, while national wealth augments in relation to the rest of the world. For example, assume that the government seeks to maintain the present value of its wealth, reinvesting all the rents and the returns from the capital invested. Taking into account capital

[9] See the discussion in Neumayer, 2010, pp. 137-41.

[10] To be sure, human capital expenditures could compensate for the depletion, but for the sake of simplicity we do not discuss this possibility here (see Hamilton, 2008; World Bank, 2011).

[11] If the natural capital under analysis does not have a market price – as in the case of many ecosystems that provide valuable productive services – or market prices do not reflect social values because of the presence of externalities– the sustainability restriction becomes much more difficult to identify.

gains, the total value of wealth at period *N* would be: $\kappa_N^G - \tilde{b}_N^G + p_N \xi_N^G = (1+\lambda)^N \left(\kappa_0^G - \tilde{b}_0^G + p_0 \xi_0^G\right)$ and fiscal redistributions would be subject to the restriction: $\sum_{t=1}^{N}(1+\lambda)^{N-t}\eta_t^D = \sum_{t=1}^{N}(1+\lambda)^{N-t}\eta_t^R$. Under the conditions of this policy, the State would become increasingly rich while simultaneously facing a tight constraint on fiscal redistributions. Furthermore, note that if $r > g$, the government income share will also grow systematically, creating a situation akin to that highlighted by Piketty (2014) with regard to capitalists. Obviously, the public sector can use the returns from capital and capital gains stemming from natural resources to finance fiscal redistributions. This fact creates a natural link between capital gains and fiscal redistributions because capital gains create fiscal space and the government has to decide whether to become richer than the private sector or to transfer the capital gains to that sector via fiscal redistributions. It would be wise, therefore, not to separate the design of fiscal accumulation and fiscal redistribution policies.

For example, the government could use not only the returns from capital but also the capital gains stemming from natural resources to finance $\eta_t^D + \eta_t^R$, which would imply the following restriction:

(27) $\sum_{t=1}^{N}(1+\lambda)^{N-t}(\eta_t^D + \eta_t^R) = (1+\lambda)^N\left(\kappa_0^G - \tilde{b}_0^G + p_0\xi_0^G\right) - \left(\kappa_0^G - \tilde{b}_0^G + p_0\xi_0^G\right)$

This policy would maintain the value of wealth at the beginning of period. That is: $\kappa_N^G - \tilde{b}_N^G + p_N\xi_N^G - \left(\kappa_0^G - \tilde{b}_0^G + p_0\xi_0^G\right) = 0$.

Note that in debt sustainability studies, the variables are typically expressed as GDP ratios while the sustainable development literature emphasizes welfare, per capita consumption and, the role of total wealth. In terms of these latter indicators, satisfying the public capital accumulation requirements of a young society going through the first stages of the demographic transition could be particularly hard. In a young society, the demand for public goods such as those that are complementary to private accumulation and urbanization will be high. Consequently, in young resource-rich countries, it is particularly relevant to consider how to spend the proceeds from natural resources. Public investment is an important determinant of the evolution of labor productivity — and, therefore, of the real wages of future workers who will have to provide for their children and retirees in the future. At the same time, a fall in public investment could easily result in a *de facto* accelerated reduction of public capital.

If the government's goal were to ensure the condition $\Delta w_t^G \geq 0$, so as to maintain at least per capita wealth ($w_t^G = \frac{W_t^G}{X_t}$)rather than the wealth/GDP ratio, ω_t^G, the resulting restriction for each period would be: $s_t^G + \hat{p}_t p_{t-1}\frac{q_{t-1}^G}{1+x} \geq x\, w_{t-1}^G$.

Where x is the rate of growth of the population. The increase in wealth is explained by both genuine savings and capital gains. The first term on the right hand side recall us that wealth accumulation must satisfy population growth.

For the government wealth to be constant, part of the total wealth should be "consumed" to finance $\eta_t^D + \eta_t^R$. If we call such a part *"z"*, the per capita government wealth will remain constant at the level w^{G^*}if:

(28) $z = \frac{r-x}{1+x}\ w^{G^*}$

This indicates that, if $r > x$, the government could allocate each year the sum z to fiscal redistribution and still respect the wealth constraint. In principle, this is independent of the path of extraction provided that $\bar{\varepsilon}_t^G = \varepsilon_t^G$ and Hotelling rules hold. The returns from capital in the period $t+1$ will increase by re_t^G, while capital gains on the existing stock of natural wealth will decrease by the same amount. Note that this differs from the usual recommendation to follow a policy such that: $s_t^G \geq 0$; under such a policy government wealth would grow systematically.

As we have already noted, the canonical approach to fiscal sustainability, which ignores government assets and natural resource depletion and pivots on equation (17), may be too restrictive to analyze some important linkages between fiscal sustainability and redistributions in developing countries that are going through the first stages of the demographic transition and/or are natural resource rich. For example, when the deficit of the social security system increases because of the aging process, there will be less space for other distributional policies unless other items of the budget adjust accordingly. Furthermore, the consequences of aging can be regressive if those covered by the social security system are richer than those that suffer the expenditure cuts or bear the increase in the tax burden. A natural resource-rich country could finance the increase in η_t^D induced by aging with rents. But, if this led to the depletion of natural resources, the policy would be unsustainable. One additional complication is that deviations from the sustainability restrictions are difficult to detect when the budget is balanced in the short run. To avoid these difficulties, the fiscal authorities should take into account the consequences of the demographic transition. We will now discuss the role of demography in more detail.

3. Fiscal Redistributions, Demography, and Wealth Constraints

This section introduces the demographic dimension and identifies the channels through which the demographic transition interacts with fiscal redistributions, sustainability and wealth.

We will use the sub index "a" to identify the different cohorts. The age of the older cohort will be $\bar{a}$. The net effect of fiscal redistributions in the case of cohort a will then be:

(29) $N_{a,t}^D = G_{a,t}^O + G_{a,t}^A + G_{a,t}^I + G_{a,t}^E + G_{a,t}^H - T_{a,t}^A - T_{a,t}^Y - T_{a,t}^I - T_{a,t}^F$

Taking this equation and the notion of life-cycle deficit as points of departure, we will show the linkages between the concepts utilized in the CEQ and NTA databases. To this end, we will first introduce the notion of aggregate life-cycle deficit and based on the life-cycle deficit and the government deficit —which are flow variables— we will subsequently study the linkages between these variables and the evolution of stocks to obtain a better grasp of the inter-temporal restrictions. The notion of life-cycle wealth is central in this regard.

The life-cycle deficit

We will define the life-cycle deficit using the concepts of 'effective consumers' and 'effective workers' that play a pivotal role in the NTA database. We will also use these two concepts to define the 'support ratio'. This indicator is utilized in the NTA literature instead of the well-known dependency ratio because it better reflects the economic consequences of the demographic transition.

To define effective consumers, the NTA approach uses the concept of a cohort's consumption ($C_{a,t}$) that includes the public provision of health, education, and other public goods. We define the ratio between cohort $a's$ per capita consumption and per capita income as: $\varphi_{a,t} = \frac{c_{a,t}}{y_t}$ and the participation of cohort a in the total population at time t as: $u_{a,t} = \frac{X_{a,t}}{X_t}$, where $X_{a,t}$ is total population of cohort a. We are now prepared to define the number of effective consumers in cohort a ($C^E_{a,t}$) as follows:

(30) $C^E_{a,t} = \varphi_{a,t}\ X_{a,t}$.

This means that the effective consumers belonging to cohort a will increase when the cohort's propensity to consume is higher. For example, because of health care expenditures, the elderly's consumption tends to be higher than average consumption. Based on this, the aggregate propensity to consume at time t (φ_t) can be disaggregated to reflect the behavior of the different cohorts. If $\bar{a}$ stands for the oldest cohort, we can write:

(31) $\varphi_t = \sum_{a=0}^{\bar{a}} \varphi_{a,t}\, u_{a,t}$

The pattern of $u_{a,t}$ will depend on the stage of the demographic transition that society is experiencing. For example, $u_{a,t}$ for $a \leq 15$ is higher in 'young' societies while the portion of the population that meets the condition $15 < a < 65$ reaches a maximum during the 'demographic bonus' stage.

$Y^L_{a,t}$ stands for the labor income of cohort a at time t and the share of cohort a in total per capita labor income ($\gamma_{a,t}$) is: $\gamma_{a,t} = \frac{y^L_{a,t}}{y_t}$. Based on the labor share of each cohort, the number of effective workers ($L_{a,t}$) is defined as:

(32) $L_{a,t} = \gamma_{a,t}\ X_{a,t}$.

The overall participation of workers in aggregate income will consequently be:

(33) $\gamma_t = \sum_{a=0}^{\bar{a}} \gamma_{a,t}\, u_{a,t}$.

As in the case of the propensity to consume, the overall labor share in income will be a function of the demographic structure via $u_{a,t}$.

In applied work, the time index of the φ and γ coefficients is usually dropped because of data limitations and the two parameters are measured only for the base year.

The non-labor income part of market income is also influenced by demographic factors. Since $Y^A_t = \sum_{a=0}^{\bar{a}} Y^A_{a,t}$, the private sector non-labor income share can be written as:

(34) $\varepsilon_t^p = \frac{y_t^A}{y_t} = \sum_{a=0}^{\bar{a}} \varepsilon_{a,t}^p u_{a,t};$

with $\varepsilon_{a,t}^p = \frac{y_{a,t}^A}{y_t}$. We can state market income as:

(35) $Y_t^M = \varepsilon_t^p Y_t + \gamma_t Y_t = Y_t(\sum_{a=0}^{\bar{a}} \varepsilon_{a,t}^p + \gamma_{a,t})u_{a,t}.$

This expression shows that via $u_{a,t}$ the demographic transition is a determinant of the labor and non-labor shares in total income.

The support ratio corresponding to cohort a is defined as the ratio between effective workers and effective consumers:

(36) $SR_{a,t} = \frac{L_{a,t}}{C_{a,t}^E}$

The support ratio is lower when the effective consumers that make up the a cohort depend more heavily on the labor income of others to finance their own consumption. If we aggregate over cohorts, society's aggregate support ratio is:

(37) $SR_t = \frac{L_t}{C_t^E} = \frac{\gamma_t}{\varphi_t} = \frac{Y_t^L}{C_t}$

This implies that the economy's support ratio increases with the participation of labor income in aggregate income and decreases with the overall propensity to consume. The expression indicates that the evolution of these two variables over time depends on behavioral and demographic factors. In young societies the support ratio is lower because of the high proportion of young individuals in the family and in old societies because of the larger share of retirees in the population. The maximum of the SR_t indicator occurs during the so-called demographic bonus stage of the demographic transition, when the conditions for growing faster improve because the proportion of the working age population in the total population reaches a maximum (Mason *et al.*, 2017)

Finally, the life-cycle deficit of the a cohort is the difference between the cohort's consumption and its labor income; expressed in per capita terms it is*:*

(38) $lcd_{a,t} = (\varphi_{a,t} - \gamma_{a,t})\, y_t.$

We can then define: $\delta_{a,t} = \frac{lcd_{a,t}}{y_t}$. In accordance with the lifecycle theory we expect this indicator to be high when a corresponds to young effective consumers or to workers that are retired, typically 60 years and older. Using the support ratio, the per capita value of the life-cycle of the cohort can be written as: $\delta_{a,t} = \varphi_{a,t}(1 - SR_{a,t})$.

The lower the cohort's support ratio is, the higher the cohort's life-cycle deficit will be. The aggregate life-cycle can be written as: $LCD_t = y_t C_t^E(1 - SR_t)$ and, consequently:

(39) $\delta_t = \sum_{a=0}^{\bar{a}} \varphi_{a,t}\big(1 - SR_{a,t}\big)\, u_{a,t}$

As can be seen, the life-cycle deficit depends on both demographic factors and the behavior of each cohort concerning consumption and the capacity to generate labor income. If $\frac{\gamma_{a,t}}{\varphi_{a,t}}$ tends to be low and $u_{a,t}$ high for young cohorts, as is the case, for example, in young societies, the aggregate life-cycle deficit will be high and will weaken the economy's ability to sustain growth. In addition, there is likely to be a strong demand for the government to finance the life-cycle deficit and this will, in turn, increase the fiscal deficit and public debt, opening the way to sustainability problems. We will now analyze the fiscal dimension of the demographic transition to show the connection between fiscal redistributions and demography.

Fiscal redistributions and cohorts

In order to introduce the fiscal dimension into the analysis so as to be consistent with the NTA methodology, it is necessary to define profiles for the 'tax burden' ($\beta_{a,t}$) and for the 'benefits received' ($\alpha_{a,t}$) by the groups. The parameter $\beta_{a,t}$ stands for per capita taxes corresponding to a given cohort normalized by per capita income and $\alpha_{a,t}$ are the transfers received by the group normalized in the same way. Hence,

(40) $\beta_{a,t} = \frac{T^A_{a,t}+T^Y_{a,t}+T^I_{a,t}+T^F_{a,t}+T^R_{a,t}}{X_{a,t}y_t}$

(41) $\alpha_{a,t} = \frac{G^O_{a,t}+G^A_{a,t}+G^I_{a,t}+G^E_{a,t}+G^H_{a,t}+G^R_{a,t}}{X_{a,t}y_t}$

As in the case of the consumption and labor income profiles, we will frequently assume that these parameters do not change because of data limitations and, hence, we will drop the t subscript and use the information corresponding to the base year. Based on the previous definitions, the aggregate tax burden and benefits as ratios of aggregate income are:

(42) $\beta_t = \sum_{a=0}^{\bar{a}} \beta_{a,t}\, u_{a,t}$

(43) $\alpha_t = \sum_{a=0}^{\bar{a}} \alpha_{a,t}\, u_{a,d,t}$

We are now prepared to study the linkages between demography and fiscal redistributions. The first step is to decompose the overall tax burden and benefits to identify fiscal redistributions. Using the superscripts D and R as we did previously:

(44) $\beta^D_t = \frac{T^A_t+T^Y_t+T^I_t+T^F_t}{Y_t}$;

(45) $\beta^R_t = \frac{T^R_t}{Y_t}$;

(46) $\alpha^D_t = \frac{G^O_t+G^A_t+G^I_t+G^E_t+G^H_t}{Y_t}$;

(47) $\alpha^R_t = \frac{G^R_t}{Y_t}$

We can then write: $\beta^D_t + \beta^R_t = \beta_t$ and $\alpha^D_t + \alpha^R_t = \alpha_t$ and, consequently:

(48) $\eta^D_t = \alpha^D_t - \beta^D_t$; and

(49) $\eta_t^R = \alpha_t^R - \beta_t^R$.

Thus, the relationship between final income and aggregate income ($\psi_t^F = \frac{y_t^F}{y_t}$) will be:

(50) $\psi_t^F = \psi_t^M + \eta_t^D = \psi_t^M + \sum_{a=0}^{\bar{a}} \alpha_{a,t}^D u_{a,t} - \sum_{a=0}^{\bar{a}} \beta_{a,t}^D u_{a,t}$,

This means that the effects of fiscal redistributions on final income will depend on the demographic profile of the economy because of the influence of $u_{a,t}$.

The ratio of the primary fiscal deficit to income ($\tilde{d}_t^{PG}$) will also be closely related to the demographic structure because, from (7): $\tilde{d}_t^{PG} = (\alpha_t^D - \beta_t^D) + (\alpha_t^R - \beta_t^R) + \iota_t^G - \varepsilon_t^G$, and consequently:

(51) $\tilde{d}_t^{PG} = \left(\sum_{a=0}^{\bar{a}} \alpha_{a,t}^D u_{a,t} - \sum_{a=0}^{\bar{a}} \beta_{a,t}^D u_{a,t}\right) + \left(\sum_{a=0}^{\bar{a}} \alpha_{a,t}^R u_{a,t} - \sum_{a=0}^{\bar{a}} \beta_{a,t}^R u_{a,t}\right) + \iota_t^G - \varepsilon_t^G$

From (21), the debt sustainability constraint can be expressed as:

(52) $\tilde{b}_0 = \sum_{t=1}^{\infty}(1+\lambda)^{-t} (\varepsilon_t^G - \alpha_t^D - \beta_t^D - \alpha_t^R - \beta_t^R - \iota_t^G)$; or:

(53) $\tilde{b}_0 = -\sum_{t=1}^{\infty}(1+\lambda)^{-t} [(\sum_{a=0}^{\bar{a}} \alpha_{a,t}^D u_{a,t} - \sum_{a=0}^{\bar{a}} \beta_{a,t}^D u_{a,t}) + (\sum_{a=0}^{\bar{a}} \alpha_{a,t}^R u_{a,t} - \sum_{a=0}^{\bar{a}} \beta_{a,t}^R u_{a,t}) + \iota_t^G - \varepsilon_t^G]$

If we interpret sustainability as a fiscal rule that sets $\tilde{b}^* = \tilde{b}_t$, at each point in time the treasury will have to respect the following restriction:

(54) $\lambda\tilde{b}^* = \tilde{b}^* \frac{r-g}{1+g} = \varepsilon_t^G - \left(\sum_{a=0}^{\bar{a}} \alpha_{a,t}^D u_{a,t} - \sum_{a=0}^{\bar{a}} \beta_{a,t}^D u_{a,t}\right) - \left(\sum_{a=0}^{\bar{a}} \alpha_{a,t}^R u_{a,t} - \sum_{a=0}^{\bar{a}} \beta_{a,t}^R u_{a,t}\right) - \iota_t^G$

And, if alternatively, the rule sets $\tilde{d}_t^{PG^*} = \lambda b_0$, the constraint will be:

(55) $\lambda b_0 = b_0 \frac{r-g}{1+g} = \varepsilon_t^G - \left(\sum_{a=0}^{\bar{a}} \alpha_{a,t}^D u_{a,t} - \sum_{a=0}^{\bar{a}} \beta_{a,t}^D u_{a,t}\right) - \left(\sum_{a=0}^{\bar{a}} \alpha_{a,t}^R u_{a,t} - \sum_{a=0}^{\bar{a}} \beta_{a,t}^R u_{a,t}\right) - \iota_t^G$

Given that these sustainability equations are, by definition, restrictions that must be respected over a long period, the demographic transition will exert its influence through the $u_{a,t}$ channel.

Wealth constraints and life-cycle wealth

Let us now analyze the way in which the demographic transition influences the evolution of the inter-temporal budget constraint of the cohorts, becoming a determinant of the path of the private and public sectors' stocks of wealth. We begin by defining the savings and wealth of the cohorts. We then aggregate those variables to obtain aggregate private savings and aggregate private wealth, and, finally, we show the linkages with the public sector's wealth constraint. In Annex 1 we illustrate the linkage between demography and wealth distribution in the case in which pensions are assumed to be deferred income.

Cohorts' savings and wealth

The a cohort's savings ($S_{a,t}$) is the difference between its final income and its consumption:

(56) $S_{a,t} = Y_{a,t}^L + Y_{a,t}^A + G_{a,t}^O + G_{a,t}^A + G_{a,t}^E + G_{a,t}^H + G_{a,t}^I + G_{a,t}^R + N_{a,t}^P - T_{a,t}^A - T_{a,t}^Y - T_{a,t}^I - T_{a,t}^R - T_{a,t}^F - C_{a,t}$

This can also be written as:

(57) $S_{a,t} = Y^F_{a,t} + N^R_{a,t} + N^P_{a,t} - C_{a,t}$

We added the variable $N^P_{a,t}$, which stands for the net transfers received by cohort a from other cohorts at time *t*. These transfers can obviously be negative and include bequests, which cannot be negative. At each point in time, private transfers have to meet the constraint: $\sum_{a=0}^{a=\bar{a}} N^P_{a,t}$= 0.

The cohort savings does not change with the inclusion of consumption in kind in overall consumption because the amount $G^E_{a,t} + G^H_{a,t} + G^R_{a,t}$ is simultaneously added to both private income and private consumption. These public expenditures items, nonetheless, reduce public savings. Taking into account fiscal redistributions and the life-cycle deficit (LCD):

(58) $S_{a,t} = Y^A_{a,t} + N^D_{a,t} + N^R_{a,t} + N^P_{a,t} - LCD_{a,t}$

Non-labor income equals the sum of returns from capital and net financial assets in the cohorts' portfolio. That is:

(59) $Y^A_{a,t} = r\, K_{a,t-1} + r\, B_{a,t-1} = r\, W^p_{a,t-1}$,

where $W^p_{a,t}$ stands for the stock of the a cohort's wealth and $K_{a,t}$and $B_{a,t}$ are the cohort holdings of capital and bonds issued by the government, respectively. To simplify, we assume a constant and equal rate of return for both assets. The "savings ratio" of the a cohort can then be defined as: $\sigma_{a,t} = \frac{S_{a,t}}{X_{a,t} y_t} = \frac{s_{a,t}}{y_t}$, or:

(60) $\sigma_{a,t} = r\, \omega^p_{a,t-1} \frac{(1+x_t)}{(1+g)(1+x_{a,t})} + \eta^D_{a,t} + \eta^R_{a,t} + \eta^P_{a,t} - \delta_{a,t}$

Where $\omega^p_{a,t} = \frac{W^p_{a,t}}{X_{a,t} y_t} = \frac{w^p_{a,t}}{y_t}$ is the a cohort's "wealth ratio"; the ratio between the a cohort's per capita wealth and aggregate per capita income and x_t and $x_{a,t}$ are the rates of growth of the total population and the population of age a, respectively. Note that: $\frac{(1+x_t)}{(1+x_{a,t})} = \frac{u_{a,t-1}}{u_{a,t}}$. Consequently, if cohort $a*$'s population is growing faster than the total population, ceteris paribus, it will be necessary to increase the cohort savings ratio to maintain the cohort's wealth ratio and this might have consequences in terms of demands for fiscal redistributions; that is, in terms of demands for changing $\eta^D_{a,t}$ in favor of cohort $a*$.

In order to calculate the evolution of the stock of wealth of each cohort over time it will be useful to identify the cohorts. We will identify the cohort according to its age at *t=0*. Therefore, $a_{0,t}$ will be the cohort of age a at *t=0*. The age of cohort a_0 at time t is equal to $a_0 + t$. The wealth at time *t* of the cohort that was a_0 years old at time *t=0* will then be denoted as $W^p_{a_0,t}$. It increases on the basis of savings: $W^p_{a_0,t} = W^p_{a_0,t-1} + S_{a_0,t}$. Given that the maximum age is $\bar{a}$, $W^p_{a_0,t} = 0$ for $a_0 + t \geq \bar{a}$. This is also true for all the budget variables corresponding to those cohorts that meet this condition.[12] If we scale per capita wealth by per capita income, the cohort's wealth evolves according to:

[12] We establish the convention that $a_0 < 0$ for the cohorts that are unborn at *t=0*. So, for example, for those who are born two years after *t=0*, the "age" at t=0 will be $a_0 = -2$. In this way, $\omega^P_{40,9}$ is the aggregate wealth of those who are 49 years old at period 9, while $\omega^P_{-2,9}$ is the wealth of those who are 7 years old at *t=9*.

(61) $\omega^{p}_{a_0,t} = \omega^{p}_{a_0,t-1}\frac{(1+x_t)}{(1+g)(1+x_{a,t})} + \sigma_{a_0,t} = \omega^{p}_{a_0,t-1}(1+\lambda^{a}_{t}) + \eta^{D}_{a_0,t} + \eta^{R}_{a_0,t} + \eta^{P}_{a_0,t} - \delta_{a_0,t})$.

Where:

(62) $(1+\lambda^{a}_{t}) = \frac{(1+r)(1+x_t)}{(1+g)(1+x_{a,t})}$.

Note that the higher $x_{a,t}$ is, the lower the growth rate of the a cohort's per capita wealth will be. This introduces a bias against social strata with higher birth rates, which are usually the poor. It follows that, to maintain the wealth ratio of these strata, fiscal redistributions should be biased in their favor. That said, to focus on the linkages between wealth dynamics and fiscal redistributions, for the moment we will make the simplifying assumption that $x_{a,t} = x_t$.Consequently, at point *t=N*, the a_0 cohort's wealth will be:

(63) $\omega^{p}_{a_0,N} = (1+\lambda)^{N}\omega^{p}_{a_0,0} + \sum_{t=1}^{N}(1+\lambda)^{N-t}(\eta^{D}_{a_0,t} + \eta^{R}_{a_0,t} + \eta^{P}_{a_0,t} - \delta_{a_0,t})$

Expressed in present value terms,

(64) $\omega^{p}_{a_0,N}(1+\lambda)^{-N} = \omega^{p}_{a_0,0} + \sum_{t=1}^{N}(1+\lambda)^{-t}(\eta^{D}_{a_0,t} + \eta^{R}_{a_0,t} + \eta^{P}_{a_0,t} - \delta_{a_0,t})$

Assuming that it is not possible to leave unpaid debts and that the members of each cohort consume or transfer to the members of other cohorts all their wealth before dying, we can write: $\omega^{p}_{a_0,N} = 0$ for $N \geq \bar{a} - a_0$. It follows that $\omega^{p}_{a_0,\bar{a}-a_0} = 0$ and, consequently,[13]

(65) $(1+\lambda)^{\bar{a}-a_0}\ \omega^{p}_{a_0,0} = -\sum_{t=1}^{\bar{a}-a_0}(1+\lambda)^{\bar{a}-a_0-t}(\eta^{D}_{a_0,t} + \eta^{R}_{a_0,t} + \eta^{P}_{a_0,t} - \delta_{a_0,t})$

This expression in present value is: $\omega^{p}_{a_0,0} = -\sum_{t=1}^{\bar{a}-a_0}(1+\lambda)^{-t}(\eta^{D}_{a_0,t} + \eta^{R}_{a_0,t} + \eta^{P}_{a_0,t} - \delta_{a_0,t})$

This implies that at time *t=0,* given the planned bequest, the higher the value of the expected stream of fiscal redistributions is, the higher the planned value of the life-cycle deficit can be. For this reason, the generosity of the social security system could negatively affect the incentives for saving. On the other hand, given the planned value for $\delta_{a_0,t}$, the value of the fiscal redistributions and of other fiscal transfers directly contribute to determining the value of the bequests that each generation will leave, which are included in: $\sum_{t=1}^{\bar{a}-a_0}(1+\lambda)^{\bar{a}-a_0-t}\ \eta^{P}_{a_0,t}$.

In sum, if any kind of transfer favors generation a_0^*, this generation will be able to run a higher life-cycle deficit and/or leave larger bequests. But we must take into account that private wealth may be unequally distributed within the a_0^* cohort. In the case of the members of the a_0 cohort who are wealthy, it is expected that both $\omega_{a_{0,0}}$ and bequests (in the form of $\eta^{P}_{a_0,t}$) will be large. If, as Piketty (2014) argues for the case of various developed countries, $r > g$ (and therefore *λ>0*), we would expect the wealth ratio ($\omega^{p}_{a_{0},0}$) of the wealthy to grow faster even if they run an elevated life-cycle deficit. We will discuss this point further in the next section.

Following the NTA terminology, we will define the present value of cohort a's "life-cycle demand for wealth" (π) at time *t=1* as:

[13] We assume that the unborn do not own assets, so wealth is also zero for $N + a_0 < 0$.

(66) $\pi_{a_0,0} = \sum_{t=1}^{\bar{a}-a_0}(1+\lambda)^{-t}\delta_{a_0,t}$, and consequently, using (65) in present value terms:

(67) $\pi_{a_0,0} = \omega_{a_0,0}^{p} + \sum_{t=1}^{\bar{a}-a_0}(1+\lambda)^{-t}(\eta_{a_0,t}^{D} + \eta_{a_0,t}^{R} + \eta_{a_0,t}^{P})$

This means that the $a_0$cohort's demand for life-cycle wealth will be satisfied with the cohort's own wealth, fiscal redistributions, and other public or private transfers, which of course could be either positive or negative. For example, in the case of the wealthy we expect bequests to be large and to take the form not only of physical and financial assets but also of transfers to finance the accumulation of human capital.

Aggregation and the macroeconomy

The evolution of the stock of wealth and of the demand for life-cycle wealth of each a_0 cohort is central to the analysis of the effects of fiscal redistributions on the allocation of wealth across generations. However, to study the consequences of the interactions between fiscal redistributions and the cohorts' behavior for macroeconomic equilibrium and the fiscal accounts, it is necessary to know the total amount of savings generated by all cohorts at each point in time and the stock of private wealth as well. We will now examine this macroeconomic dimension.

The first step will be to obtain an expression for the overall propensity to save (σ_t^P). To that end, we have to aggregate the savings of all cohorts. Considering that the sum of private transfers adds up to zero, we can write:

(68) $\sigma_t^P = \frac{S_t^P}{Y_t} = \sum_{a=0}^{\bar{a}}\left(r\omega_{a,t-1}^{p}\frac{(1+x_t)}{(1+g)(1+x_{a,t})} + \eta_{a,t}^{D} + \eta_{a,t}^{R} - \delta_{a,t}\right)u_{a,t}$

(69) $\sigma_t^P = \frac{r\omega_{t-1}^{P}}{1+g} + \eta_t^D + \eta_t^R - \delta_t$

According to (68), the overall propensity to save is a function of fiscal redistributions and the demographic transition. Demography operates through three channels. The first is the private sector's behavioral profiles associated with non-labor income ($r\omega_{a,t}^{p}$) and the life-cycle deficit ($\delta_{a,t}$). These variables are a function of the cohort's age, the behavior concerning bequests, and the features of the social security system. These two latter factors influence the incentives to accumulate wealth. The second has to do with fiscal redistributions, which also change according to age and the features of social protection policies, taxes, and the social security system. The third is the weight of each cohort's population in the total population, which is expected to change as the economy goes through the different stages of the demographic transition.

The overall private sector wealth constraint at time *t* results from the aggregation of the individual wealth of all cohorts:

(70) $\omega_t^P = \frac{W_t^P}{Y_t} = \sum_{a=0}^{\bar{a}}(\frac{\omega_{a,t-1}^{p}}{(1+g)}u_{a,t-1} + \sigma_{a,t}u_{a,t}) = \frac{\omega_{t-1}^{P}}{1+g} + \sigma_t^P$

Using this expression and taking into account that we assume that the private sector does not own natural resources, we can state the aggregate savings ratio as:

(71) $\sigma_t^P = \dfrac{r\kappa_{t-1}^P + r\tilde{b}_{t-1}^P}{(1+g)} + \eta_t^D + \eta_t^R - \delta_t$

where kappa, the private "capital/output ratio", is defined as: $\kappa_t^P = \frac{k_t^P}{y_t}$ and $\tilde{b}_t^P = \frac{b_t^P}{y_t}$ are the public bonds held by all cohorts in relation to GDP. We can see the way in which fiscal redistributions enter the picture if we write the aggregate wealth ratio in terms of savings components:

(72) $\omega_t^P = \sum_{a=0}^{\bar{a}}[(1+\lambda)\omega_{a,t-1}^p u_{a,t-1} + (\eta_{a,t}^D + \eta_{a,t}^R - \delta_{a,t})u_{a,t}]$

As was mentioned above, $u_{a,t}$ is expected to change substantially over the demographic transition, changing the group's wealth ratios. The aggregate private sector's wealth ratio can be written more synthetically as:

(73) $\omega_t^P = \frac{(1+r)}{1+g}\omega_{t-1}^P + \eta_t^D + \eta_t^R - \delta_t$;

and: $\kappa_t^P + \tilde{b}_t^P = (1+\lambda)\left(\kappa_{t-1}^P + \tilde{b}_{t-1}^P\right) + \eta_t^D + \eta_t^R - \delta_t$

Solving this difference equation:

(74) $\kappa_N^P + \tilde{b}_N^P = (\kappa_0^P + \tilde{b}_0^P)(1+\lambda)^N + \sum_{t=1}^{N}(1+\lambda)^{N-t}(\eta_t^D + \eta_t^R - \delta_t)$

In present value terms:

(75) $(\kappa_N^P + \tilde{b}_N^P)(1+\lambda)^{-N} = \kappa_0^P + \tilde{b}_0^P + \sum_{t=1}^{N}(1+\lambda)^{-t}(\eta_t^D + \eta_t^R - \delta_t)$

Assuming rationality and the non-Ponzi game condition, and taking $\lim\limits_{N\to\infty}$, we have:

(76) $\kappa_0^P + \tilde{b}_0^P = \sum_{t=1}^{\infty}(1+\lambda)^{-t}(\delta_t - \eta_t^D - \eta_t^R)$

The aggregate wealth of the private sector must equal the present value of the life-cycle deficit net of public interventions.

And in terms of the aggregate private sector's life-cycle wealth (π)

(77) $\pi_0 = \kappa_0^P + \tilde{b}_0^P + \sum_{t=1}^{\infty}(1+\lambda)^{-t}(\eta_t^D + \eta_t^R)$

Given $\omega_0^P = \kappa_0^P + \tilde{b}_0^P$, the greater π_0 is, the higher the cohort's demand for public transfers and the provision of public goods will be. In particular, in an aging society we expect π_0 to be higher than in a "young" one. Consequently, if accumulated private wealth is low and the society is aging, there will be an increasing pressure on η_t^D via the social security system and a substantial trade-off between η_t^D and η_t^R might arise. In any case, there is likely to be pressure on public primary expenditures and the primary deficit. However, beyond the financial difficulties associated with a higher deficit, it will not be possible to satisfy the private sector's demands if accumulated wealth is insufficient. In order to analyze this point we have to introduce the government's wealth constraint.

If the markets for bonds ensure that

(78) $\tilde{b}_t = \tilde{b}_t^P$

we can state the national aggregate wealth ratio for the case of the closed economy (ω) as:

(79) $\omega_0 = \kappa_0^P + \kappa_0^G + p_0\xi_0^G = \sum_{t=1}^{\infty}(1+\lambda)^{-t}\delta_t = \pi_0$

That is, beyond public redistributions, existing national wealth should suffice to cover the present value of the stream of future life-cycle deficits. Note that: $\kappa_0^G + p_0\xi_0^G = \pi_0 - \kappa_0^P$, which means that public wealth is used to finance the portion of the demand for life-cycle wealth that cannot be covered by private wealth. Hence, when publicly-owned enterprises exploit natural resources, such resources can easily be consumed in a non-optimal and/or inequitable way if the social security system is ill-designed. Note that if $\hat{p}_t > 0$ ($\hat{p}_t < 0$), the public sector will have a net gain (loss) of wealth, which will be symmetrical to the loss (gain) of the private sector. Since there is no wealth creation, the overall restriction (79) still holds. If we included the rest of the world, however, there would be a net gain (loss) for the national economy.

4. Fiscal Redistributions and Income Strata

So far, we have analyzed the budgetary consequences of fiscal redistributions without distinguishing between income strata. The main purpose of this section is to include the income strata in the methodological framework. We will identify the strata with the subscript "d", which can be interpreted, for example, as deciles or quintiles, or low income and high income. The effect of fiscal redistributions for the case of income group "d" at time t ($N_{d,t}^D$) will be:

(80) $N_{d,t}^D = G_{d,t}^O + G_{dt}^A + G_{d,t}^I + G_{d,t}^E + G_{d,t}^H - T_{d,t}^A - T_{d,t}^Y - T_{d,t}^I - T_{d,t}^F$

Whether the sign of $N_{d,t}^D$ is positive or negative for a specific stratum is central to assessing the overall impact of redistributions. If $N_{d^*,t}^D < 0$, the stratum d^* will be contributing to finance aggregate fiscal redistributions . This means that the final income will be higher for those income groups that benefit from fiscal redistributions and lower for the groups that contribute to financing such policies because: $Y_{d.t}^F = Y_{d,t}^M + N_{d,t}^D$. In order to assess the impact of specific fiscal redistributions, a common strategy is to compare the value of the Gini coefficient before the intervention (i.e., calculated on the basis of $Y_{d,t}^M$) with the value of that coefficient after the intervention (i.e., calculated on the basis of $Y_{d.t}^F$). If the former is higher, we can say that the public redistribution is progressive.

If the total number of strata is $\bar{d}$, it is possible to implement a policy for which $N_{d,t}^D \geq 0;\ \forall d$ and, therefore, $\sum_{d=1}^{\bar{d}} N_{d,t}^D > 0$. One would expect that this type of redistribution would face less resistance, but it could have negative effects on fiscal sustainability or distortionary effects on the allocation of the fiscal space, as we have already discussed. Additionally, it is important to evaluate the incidence of all the components included in $N_{d,t}^D$, especially when the policy seeks to protect the poor or can impinge differently on distinct cohorts. For example, protection policies that are targeted to curb poverty could be judged to be satisfactory because in the case of those below the poverty line, $G^O_{d,t} + G^A_{dt} + G^I_{d,t} + G^E_{d,t} + G^H_{d,t}$>0. But when the incidence of indirect taxes is high on the poor (particularly consumption taxes), $N^D_{d,t}$ might become negative for those individuals with an income that falls below the poverty line, which means that the poor will contribute to financing fiscal redistributions in net terms[14]. And this may occur even if the fiscal intervention is progressive in the sense that $N^D_{d,t} < 0$ for the highest income levels.

We will maintain the convention of using lowercase letters for variables expressed in per capita terms ($X_{d,t}$ will stand for the number of people in the group under consideration) and Greek letters for ratios with respect to aggregate income. Therefore, for example, $\psi^M_{d,t}$ will be the pre-tax share of group *d*'s per capita income in aggregate per capita income, which is equal to the sum of the participation of the group's per capita labor income ($\gamma_{d,t}$) and non-labor income ($\varepsilon^p_{d,t}$) in aggregate per capita income, that is: $\psi^M_{d,t} = \frac{y^M_{d,t}}{y_t} = \frac{y^L_{d,t}+y^A_{d,t}}{y_t} = \gamma_{d,t} + \varepsilon^p_{d,t}$.

A straightforward way to compare the effects of the set of existing fiscal redistributions on the market incomes of two specific groups is to calculate the ratio of the income shares of the two groups before and after fiscal redistributions. More specifically, consider two groups: high income (*d=h*) and low income (*d=l*). We can say that the existing fiscal redistributions favor group l over group h if $\frac{\psi^M_{h,t}}{\psi^M_{l,t}} > \frac{\gamma_{h,t}+\epsilon^p_{h,t}+\eta^D_{h,t}}{\gamma_{l,t}+\epsilon^p_{l,t}+\eta^D_{l,t}}$. This means that both the labor share and asset owners' share corresponding to the groups involved is likely to be affected by fiscal redistributions. Of course, the consequences in terms of incentives to invest and work will be different depending on the effects of fiscal redistributions on the workers' or the capital owner's shares. Note that income distribution could worsen even though fiscal redistributions are very effective. This could happen if the income share of group h is increasing in relation to group l. Indeed, Stiglitz (2015) identifies a set of new stylized facts that requires explanation, and one of them is that the labor share is worsening. This is also compatible with the facts raised by Piketty (2014).

If $u_{d,t}$ is the share of the total population accounted for by group d, *(*$\frac{X_{d,t}}{X_t}$*)*, and $\eta_{d,t} = \frac{n^D_{d,t}}{y_t}$ is the ratio between per capita fiscal redistributions and per capita income corresponding to group d, we can write:

(81) $\eta^D_t = \frac{N^D_t}{Y_t} = \sum_{d=1}^{\bar{d}} \eta^D_{d,t}\, u_{d,t}$.

Once we calculate the aggregate value η^D_t, using (7) we can obtain the value of the fiscal deficit ($\tilde{d}^G_t$) and show the linkages with income distribution:

(82) $\tilde{d}^G_t = r\tilde{b}_{t-1} + \sum_{d=1}^{\bar{d}} \eta^R_{d,t}\, u_{d,t} + \sum_{d=1}^{\bar{d}} \eta^D_{d,t}\, u_{d,t} + \iota^G_t - \varepsilon^G_t$.

And it is possible to assess the macroeconomic consequences in terms of fiscal sustainability using (21):

(83) $\tilde{b}_0 = \sum_{t=1}^{\infty}(1+\lambda)^{-t}\left(\sum_{d=1}^{\bar{d}} \eta^R_{d,t}\, u_{d,t} + \sum_{d=1}^{\bar{d}} \eta^D_{d,t}\, u_{d,t} + \iota^G_t - \varepsilon^G_t\right) = -\sum_{t=1}^{\infty}(1+\lambda)^{-t}\, \tilde{d}^{PG}_t$

[14] On this issue see Lustig et al. (2014).

These expressions show that the overall incidence of the fiscal redistributions as a share of per capita income (η_t^D) depends on both, the incidence on the per capita income and the size of each group. For example, the case may be that η_t^D >0 is sizable – which means that overall fiscal redistributions use a significant part of the fiscal space – while for the lowest income strata $\eta_{l,t}^D$ >0 but the size of the per capita transfer is meager because $u_{l,t}$is large, which is the case in "young" societies. This could be a difficult situation if the tax base is reduced because evasion or elusion is pervasive, or because tax revenues are falling due to aging. Under such circumstances, the tax pressure on those who pay taxes will be too high while the benefits per capita received will be too low. Fiscal sustainability, in turn, could be at risk if initiatives to increase $\eta_{l,t}^D$ were implemented in circumstances in which access to credit is becoming difficult.

However, similar situations can occur because of perverse interactions between demographic factors and the features of the social security system – or when volatile rents of natural resources have an important role in generating fiscal resources to finance $\Delta\eta_t^D$ >0. For example, suppose that during the boom $\Delta\eta_t^D = \Delta\varepsilon_t^G$. If the positive shock is permanent, fiscal sustainability will not be affected. But if it is transitory, to meet the sustainability condition $\lambda b_0 = -\tilde{d}_t^{GP}$, fiscal redistributions should return to their previous values and the burden of the adjustment could be distributed in a more regressive way among the groups because of the urgencies of fiscal adjustment.

If the change in the structure of fiscal redistributions is not large, we can state:

(84) $\Delta\eta_t^D = \sum_{d=1}^{\bar{d}} \Delta(\eta_{d,t}^D u_{d,t}) \cong \sum_{d=1}^{\bar{d}} (\Delta\eta_{d,t}^D u_{d,t} + \Delta u_{d,t}\eta_{d,t}^D)$

Note that demographic dynamics enter naturally into the analysis through the variable $\Delta u_{d,t}$. Even if the fiscal authorities established $\Delta\eta_{d,t}^D = 0$ in order to maintain the distributional status quo, $\Delta\eta_t^D$ would not be zero if the participation of each group in total population were changing. For example, this is the case of a State that experiences a mounting fiscal deficit and faces sustainability risks as a consequence of aging, as we have mentioned above. On the other hand, if the condition to be met were $\Delta\eta_t^D = 0$ in order to avoid budgetary imbalances, the changes in $\eta_{d,t}^D$ would have to compensate each other to offset the effects of budgetary changes: $\sum_{d=1}^{\bar{d}} \Delta\eta_{d,t}^D u_{d,t} \cong \sum_{d=1}^{\bar{d}} \Delta u_{d,t}\eta_{d,t}^D$. In this case if the adjusting variable is, say, the VAT rate, the impact on the poor could be disproportionate.

The disaggregation of fiscal redistributions according to strata may help detect "perverse" distributional effects associated with the existing structure of fiscal redistributions. Consider an economy with a significant presence of informal labor markets. The pension system may also be a source of regressive fiscal redistributions. For example, this might easily happen if the main "redistributive mechanisms" are pension transfers that cover those retired workers who worked in the formal sector in a society that is undergoing the earlier stages of the demographic transition in which poverty is particularly high among the youngest.

Integrating distribution and demography

We will now consider simultaneously the linkages between fiscal redistributions, on the one hand, and cohorts and income strata, on the other. We will provide some examples that are relevant to the processes of structural change that usually accompany development and demographic transitions. The purpose is to highlight the relevance of having a greater availability of data that considers demography and income strata simultaneously.

We can identify the groups under analysis on the basis of the subscripts that we were using for cohorts and income strata, that is, a and d. Indeed, the main limitation in this regard is not methodological but rather data availability. Nevertheless, it is possible to use the methodology to perform simulation exercises based on partial information and educated guesses.

The net effect of the fiscal redistribution on the market income of the group of income level d and age a will be:

(85) $N^D_{a,d,t} = G^O_{a,d,t} + G^A_{a,d,t} + G^I_{a,d,t} + G^E_{a,d,t} + G^H_{a,d,t} - T^A_{a,d,t} - T^Y_{a,d,t} - T^I_{a,d,t} - T^F_{a,d,t}$

and, consequently, $\eta^D_{a,d,t} = \frac{n^D_{a,d,t}}{y_t}$ will stand for the ratio of per capita fiscal redistributions to per capita income corresponding to group of age a and stratum d.

The behavioral parameters that we need for the basic demographic notions must be redefined accordingly. Therefore, φ_t, for example, will have to be disaggregated to reflect the behavioral profiles of the different a,d groups: $\varphi_{a,d,t} = \frac{c^p_{a,d,t}}{y_t}$, and, therefore[15],

(86) $\varphi_t = \frac{C^P_t}{Y_t} = \sum_{a=0}^{\bar{a}} \sum_{d=1}^{\bar{d}} \varphi_{a,d,t}\ u_{a,d,t}$

The parameter $\varphi_{a,d,t}$ is the share of the economy's overall propensity to consume corresponding to the group a,d at time t. Under these conditions, the overall propensity to consume at time t will be a function of both the demographic structure and the distribution of consumption among income strata because it depends on $u_{a,d,t}$ and $\varphi_{a,d,t}$.

Following the same logic, the labor income of the a,d group $(Y^L_{a,d,t})$ can be aggregated to obtain the aggregate labor income: $Y^L_t = \sum_{a=0}^{\bar{a}} \sum_{d=1}^{\bar{d}} Y^L_{a,d,t}$. The share of group a, d labor income in total per capita labor income is: $\gamma_{a,d,t} = \frac{y^L_{a,d,t}}{y^M_t}$ and, therefore, the overall participation of labor in aggregate income will be:

(87) $\gamma_t = \frac{Y^L_t}{Y_t} = \sum_{a=0}^{\bar{a}} \sum_{d=1}^{\bar{d}} \gamma_{a,d,t}\, u_{a,d,t}$.

The non-labor income part of market income, in turn, will be: $Y^A_t = \sum_{a=0}^{\bar{a}} \sum_{d=1}^{\bar{d}} Y^A_{a,d,t}$, and then it follows that:

[15] The overall propensity to consume can also be expressed as: $\varphi_t = \sum_{a=0}^{\bar{a}} \varphi_{a,t}\, \mu_{at} = \sum_{d=1}^{\bar{d}} \varphi_{d,t}\, u_{dt}$ where: $\varphi_{a,t} = \sum_{d=1}^{\bar{d}} \varphi_{a,d,t} \frac{x_{a,d,t}}{x_{a,t}}$ and $\varphi_{d,t} = \sum_{a=0}^{\bar{a}} \varphi_{a,d,t} \frac{x_{a,d,t}}{x_{d,t}}$.

(88) $\varepsilon_t^p = \frac{Y_t^A}{Y_t}$=$\sum_{a=0}^{\bar{a}} \sum_{d=1}^{\bar{d}} \varepsilon_{a,d,t}^p u_{a,d,t}$;

with $\varepsilon_{a,d,t}^p = \frac{y_{a,d,t}^A}{y_t}$. One important implication of (93) and (94) is that labor and non-labor shares are not independent of the demographic transition because of the influence of $u_{a,d,t}$.

Once we have $\varphi_{a,d,t}$, ε_t^p and $\gamma_{a,d,t}$, it is possible to calculate the main demographic indicators – effective consumers, effective workers, and the life-cycle deficit – and to show the connections between them, on the one hand, and $\eta_{a,d,t}^D$,the fiscal deficit, sustainability and wealth, on the other. In order to do so we must proceed as we have done in the previous sections. It is also possible to evaluate the evolution of wealth.

The methodology may also help to call attention to the role of the labor market when interpreting the evolution of demographic variables in a process of development and structural change. The following two indicators are useful in this regard. The first is "employment intensity": $f_{a,d,t} = \frac{X_{a,d,t}^e}{X_{a,d,t}}$, where $X_{a,d,t}^e$ stands for the members of the a,d group that are employed. The second is a "wage correction factor" $\tilde{v}_{a,d,t} = \frac{v_{a,d,t}}{v_t}$, which shows the relation between the a,d group average wage ($v_{a,d,t}$) and the average wage of the economy (v_t). Using these variables, the per capita labor income of the a,d group can be expressed as: $y_{a,d,t}^L = \tilde{v}_{a,d,t} v_t \, f_{a,d,t}$. The correction factor $\tilde{v}_{a,d,t}$ reflects wage differences between groups determined by disparities in human capital accumulation, labor market failures, and age-related factors, such as experience or the ability to work.

The variable $f_{a,d,t}$ can help examine many factors. First, the economy's ability to create jobs for the different a,d groups. Second, owing to the influence of demographic factors, we expect $f_{a,d,t}$ to vary significantly across cohorts. But for a given cohort a, we also expect the employment intensity to be correlated with the income level, which in turn tends to be associated with access to the labor market. Third, the social security institutions that determine the retirement date will have a bearing on $f_{a,d,t}$ in the case of older cohorts. Monitoring the evolution of $f_{a,d,t}$ is critical if we take into account that being employed helps to reduce the probability of being poor and that unemployment is typically high in the case of some cohorts that are in a vulnerable position, as in the case of the young.

On the other hand, a favorable evolution of $f_t = \sum_{a=0}^{\bar{a}} \sum_{d=1}^{\bar{d}} f_{a,d,t} \, u_{a,d,t}$ can be interpreted as an indicator of "positive" overall structural change because it tells us whether employment is increasing faster than the working-age population. If the economy is able to maintain $\Delta f_t > 0$ for a sustained period, the growth process will probably be accompanied by the reduction of unemployment and informality, increasing women's participation and reductions in the share of population below the poverty line. During the so-called demographic bonus – when the growth rate of the working-age population is expected to reach a maximum – it is important that $\Delta f_t \geq 0$. However, a situation in which: $\Delta \, f_t \leq 0$ may easily occur if the investment rate in physical and human capital is low and it could have deleterious consequences for both inequality and poverty. The income share of the a,d group will increase if the group is doing well vis-à-vis job creation $\Delta f_{a,d,t} > \Delta f_t$ or its human capital endowment is improving fast (and, hence, $\Delta \tilde{v}_{a,d,t}$ is high). These factors can make a particularly important contribution to equity if the dependency rate corresponding to the group is high. Finally, a dynamic evolution of Δf_t can be a blessing for fiscal sustainability because new workers produce income and consume increasing tax revenues, and contribute to financing the social security system, depending on the degree of informality in the labor market. Indeed, it is because of the fact that $\Delta f_t > 0$ over a long period that the so-called demographic window of opportunity, as well as the reduction in economic duality, can contribute to accelerating growth.

5. Concluding remarks

In this chapter we developed a methodological framework to study the linkages between fiscal redistributions, fiscal sustainability and the government's wealth constraint. We included demographic factors and income strata and underscored the importance of increasing the availability of data that considers demographic and distributional features simultaneously. We made an effort to show the connections between the NTA and CEQ concepts and suggested possible synergies and directions for further data collection efforts. We also tried to illustrate the implications of the framework in terms of the research agenda on development. In particular, we underscored the importance of the analysis of fiscal sustainability including all public assets in the government's balance sheet, especially natural resources. This is functional to connecting "the two views of sustainability": the one concerning fiscal soundness and the one concerning development. We believe, in this regard, that more research is needed, first, about the role of public wealth including all assets in the government's balance sheet and about the distributional consequences – on income as well as wealth – of policies regarding fiscal sustainability, intergenerational transfers that finance the demand for life-cycle wealth, and the management of publicly-owned natural resources.

We have paid special attention to the case of natural resource-rich developing countries that are going through the first stages of the demographic transition or are enjoying the demographic window of opportunity. One issue that requires more research work is volatility. When international prices of natural resources in such countries rise substantially, political forces are likely to press for an increase in N_t^D because of the increment in rents revenues, E_t^G. But prices are volatile and shocks are, more often than not, transitory. Therefore, in the phase in which prices drop, the primary balance will likely worsen and the previous increases in redistribution policies, N_t^D, will ultimately result in a higher public debt/GDP ratio. Demography and wealth constraints, in turn, enter the picture because greater indebtedness means that the funds to finance fiscal redistributions will be provided by different cohorts, which are not typically favored in the same way by the increase in redistribution policies. The longer the duration of public debt is, the more probable it is that significant inter-generational distributions will be involved (see Fanelli, 2015). Besides, we have shown the significance of the additional burden for each cohort will depend not only on the increase in the debt ratio but also on the size of each of the cohorts, the growth rate of the economy, and the proportion of tax payers and beneficiaries of public spending in each cohort, which, in turn, will be a function of the stage of the demographic transition. On the other hand, the specific combination of generations' debt burden and debt duration features will influence the market perception of the maximum level of the debt ratio that is considered sustainable and, hence, the government's ability to access credit markets. In order to reduce D_t^{PG} in a period of reduced revenues from rents and weakening fiscal sustainability the government might try to increase E_t^G by increasing extraction (ΔQ_t^G). This, in turn, would result in a faster depletion of non-renewable resources, making growth less sustainable.

On the basis of these issues, the following policy implications of the methodological framework deserve mention.

(1) Fiscal sustainability tests should be part of any significant initiative involving fiscal redistributions. Policies that do not pass the sustainability tests could undermine the ability of the state to improve income distribution, protect the poor and create a growth friendly environment over time, giving rise to socially disruptive phenomena, such as "lost decades".

(2) In the case of natural resource-rich countries it is particularly relevant to consider that fiscal redistributions may ultimately deplete the stock of natural resources without ensuring a compensatory accumulation of reproducible capital if they do not take adjusted government savings and capital gains into account.

(3) In the context of the ongoing demographic transition, even if the parameters of fiscal redistributions are maintained, the changes in the weight of the different cohorts in the total population will modify the size of fiscal redistributions. This is one important reason why demography must be taken into consideration when designing fiscal redistributions and assessing sustainability. It also matters to income and wealth distribution to the extent that income distribution differs among cohorts.

(4) Transfers associated with the social security system are a substantial part of public redistributions and a key determinant of both the life-cycle deficit and the government deficit. The ways in which life-cycle deficits and the demand for the life-cycle wealth of each cohort are financed impinge significantly on the distribution of wealth between the public and the private sectors and across-generations as well.

(5) Whether we consider pensions as deferred income or not matters for the distribution of wealth between the public and private sectors and, probably, for public opinion's perception of the significance of public redistributions. Public opinion's misperception can easily result in a demand for life-cycle wealth that cannot be satisfied, given the economy's capacity to accumulate wealth and the restrictions imposed by sustainable development on the trajectory of natural capital.

Annex 1

Pensions as deferred income

The relation between the social security system and fiscal redistributions raises the issue of whether contributory pensions should be considered a form of fiscal redistribution or deferred incomes. We will now investigate the consequences of conceiving pensions as deferred income. We begin by adding the contributory social insurance old-age pensions net of subsidies ($G_t^{A'}$) to market income. In this way we obtain a "corrected" version of market income ($Y_t^{M'}$). Two clarifications are in order. First, $G_t^{A'}$ is not considered a transfer from the public to the private sector but is the perception of deferred income by the private sector. For this reason we do not include the subsidized part of pension transfers (Z_t^A) included in G_t^A, if any. This means, of course, that: $G_t^A = G_t^{A'} + Z_t^A$. Second, when adopting this perspective, social security contributions (T_t^A) must be assumed to be mandatory savings. Corrected market income is then:

(A1) $Y_t^{M'} = Y_t^L + Y_t^A + G_t^{A'}$

To calculate corrected disposable income ($Y_t^{D'}$) contributions to social security are not deducted from labor earnings because, as was already mentioned, they are assumed to be mandatory savings. But we have to add the subsidized part of pensions to contributory pensions. Consequently:

(A2) $Y_t^{D'} = Y_t^L + Y_t^A + G_t^O + G_t^{A'} + Z_t^A - T_t^Y$

Corrected consumable income ($Y_t^{C'}$) is, then:

(A3) $Y_t^{C'} = Y_t^L + Y_t^A + G_t^O + G_t^{A'} + Z_t^A + G_t^I - T_t^Y - T_t^I$,

while corrected final income can be stated as:

(A4) $Y_t^{F'} = Y_t^L + Y_t^A + G_t^O + G_t^{A'} + Z_t^A + G_t^I + G_t^E + G_t^H - T_t^Y - T_t^I - T_t^F$.

This means that the corrected version of disposable, consumable, and final income will be higher than their non- corrected counterparts by an amount equal to $T_t^A + G_t^{A'} + Z_t^A - G_t^A = T_t^A$.

We should correct the expression for the distributive effects of fiscal redistributions in accordance with the new assumptions. In particular, there is now no redistribution via fiscal intervention concerning pensions, with the exception of their subsidized share. Therefore, we define:

(A5) $N_t^{D'} = G_t^O + G_t^I + G_t^E + G_t^H + Z_t^A - T_t^Y - T_t^I - T_t^F$

The relation between the two concepts, then, is:

(A6) $N_t^D = N_t^{D'} + G_t^A - Z_t^A - T_t^A = N_t^{D'} + G_t^{A'} - T_t^A$.

This means that the fiscal and redistributive effects that will be attributed to fiscal redistributions under the assumption of deferred income may greatly differ from the effects under standard assumptions. The difference depends on the value of $G_t^{A'} - T_t^A$ and the distribution of $G_t^{A'}$ and T_t^A among income strata. The greater the subsidized part of contributory pensions is, the lower the difference between N_t^D and $N_t^{D'}$ will be. Likewise, the difference will be low when the deficit of the social security system ($G_t^A - T_t^A$) is low.

When we look at this issue from an inter-temporal point of view, a conceptual discussion is in order. We are assuming that contributory pensions originate in deferred income when in reality social security follows a pay-as-you-go rule. If we cease to record current contributions as part of public revenues but the government still has to pay committed pensions, the current and future primary fiscal deficits will obviously increase. Since $G_t^{A'}$ is assumed to be the return to assets that were previously accumulated by the private sector on the basis of T_{t-m}^{A}, with *m>0*, the present value of the future fiscal deficits associated with those payments must be added to the existing public debt. Three issues deserve mention. First, any future pension payment in excess of the normal returns on an annuity that the pensioner could buy in the markets with the funds accumulated at the date of her retirement should be considered a subsidy and imputed to Z_t^A. In other words, the payments that will be received in the annuitization phase should only reflect the amount of income that was deferred in the accumulation phase and any payment beyond this is a subsidy. Second, the costs of the increasing longevity risk that a defined payment pension system faces as a consequence of aging should also be considered a subsidy. Third, if we cease recording T_{t+m}^{A} in the government budget, the private sector will have to cover an increasing part of future pensions due to the fact that, from *t=0* onwards, they will begin to accumulate stocks in their portfolios based on their forced savings, which equals T_{t+m}^{A} at each period $m \geq 0$. The assets acquired with forced savings should be used, in turn, to buy the annuities at the date of retirement. In this sense, to consider contributory pensions as deferred income is analogous to simulating what the budgetary consequences would be if the government reformed social security and replaced the pay-as-you-go system with one based on the private capitalization of individual contributions. The reform experiences show that at the moment of the implementation of the reform (*t=0)* there is a substantial increase in the primary deficit, which subsequently and gradually disappears. That is, $G_t^{A'}$ shows a downward trend while the assets accumulated in private portfolios on the basis of forced savings, on the contrary, show an upward trend. Consequently, from *t=0* onwards, only the falling magnitude $G_{0+m}^{A'}$; $m \geq 0$ should be recorded in the budget.

To perform sustainability exercises we need to create a public liability that reflects the present value of the stream of future $G_t^{A'} + Z_t^A$ payments from *t=0* onward. Taking into account that Z_t^A is already included in $N_t^{D'}$, and that this variable excludes T_t^A, the inter-temporal restriction will be:

(A7) $\tilde{b}_0 = \sum_{t=1}^{\infty}(1+\lambda)^{-t}\left(\varepsilon_t^G - \eta_t^{D'} - \frac{g_t^{A'}}{y_t} - \eta_t^R - \iota_t^G\right) = -\sum_{t=1}^{\infty}(1+\lambda)^{-t}\,(\tilde{d}_t^{PG} + \frac{t^A}{y_t})$

Where $\sum_{t=1}^{\infty}(1+\lambda)^{-t}(\frac{g_t^{A'}}{y_t} + \frac{z_t^A}{y_t})$ is the present value of the government liability generated by pension transfers. The liabilities will increase the overall debt burden because the government ceases to receive social security contributions ($\frac{t^A}{y_t}$). If the fiscal rule is $\tilde{d}^{PG^*} = -\lambda\tilde{b}_0$, the restriction on $\eta_t^{D'}$ will be:

(A8) $\eta_t^{D'} = \varepsilon_t^G - \tilde{g}^{A'} - \eta_t^R - \iota_t^G - \lambda\tilde{b}_0$

And, under the more usual sustainability condition $\tilde{b}_t = \tilde{b}^*$, at each point in time the government will have to respect the restriction: $\eta_t^{D'} = \varepsilon_t^G - \tilde{g}^{A'} - \eta_t^R - \iota_t^G - \lambda \tilde{b}^*$.

To be sure, the public sector may have accumulated non-financial assets using previous contributions to social security. But it is very difficult to identify those assets because, for one thing, public assets are only partially registered, and their market value is difficult to assess and, for another, more often than not public accounting does not register what assets – if any – are acquired with the proceeds from social security.

Income strata and deferred income

The deferred income assumption may help uncover hidden subsidies. The following example may clarify the issue. Since the difference between $N_{d,t}^D$ and $N_{d,t}^{D'}$ originates in the treatment of contributory pensions, the relevance of such difference is a direct function of the proportion of the population older than 65 years in the income group. Note the $N_{d,t}^D - N_{d,t}^{D'}$ gap. If we consider that pensions are deferred income and group l is favored over group h by fiscal redistributions, it means that $N_{h,t}^{D'} < N_{l,t}^{D'}$. But this is compatible with a situation in which: $N_{l,t}^{D'} - N_{h,t}^{D'} < \left(G_{h,t}^{A'} - T_{h,t}^A\right) - (G_{l,t}^{A'} - T_{l,t}^A)$. Under these circumstances, the net benefits that group h receives from social security more than compensates for the group's disadvantageous position concerning other transfers and taxes and the group becomes a net winner. If $G_{h,t}^{A'}$ does in fact have a large component of hidden subsidy not registered in $Z_{h,t}^A$, the situation will not be equitable. For example, let us assume that the l group is composed basically of young people and the average age of the h group is much higher. The positive effect of the conditional transfers favoring the young may be more than offset by the effect of pension transfers. If the older are richer than the younger, the overall result is regressive. This type of outcome can be seen in Latin America.

Private wealth and forced savings

If we consider T^A as "forced savings", the present value of forced savings is part of the private sector wealth. Therefore, we can write:

$$\text{(A9)} \quad \kappa_0^P + \tilde{b}_0^P + \sum_{t=1}^{\infty}(1+\lambda)^{-t}\frac{T_t^A}{X_t y_t} = \sum_{t=1}^{\infty}(1+\lambda)^{-t}\left(\delta_t - \eta_t^{D'} - \frac{G_t^{A'}}{X_t y_t} - \eta_t^R\right)$$

Consequently, forced savings contribute to financing the demand for life-cycle wealth.

$$\text{(A10)} \quad \pi_0 = \kappa_0^P + \tilde{b}_0^P + \sum_{t=1}^{\infty}(1+\lambda)^{-t}(\eta_t^{D'} + \frac{G_t^{A'}}{X_t y_t} + \eta_t^R + \frac{T_t^A}{X_t y_t})$$

In turn, we have to subtract the present value of the contributions to the social security system from the fiscal budget. Given that the committed pension payments Z_t are already registered in $\eta_t^{D'}$, we can write:

$$\text{(A11)} \quad \kappa_0^G - \tilde{b}_0 - \sum_{t=1}^{\infty}(1+\lambda)^{-t}\frac{T_t^A}{X_t y_t} = \sum_{t=1}^{\infty}(1+\lambda)^{-t}\left(\eta_t^{D'} + \frac{G_t^{A'}}{X_t y_t} + \eta_t^R\right)$$

Demography and wealth

In terms of pensions as deferred income, $Y_t^{F'} = Y_t^M + G_t^{A'} + N_t^{D'}$ and, consequently, corrected private savings is:

(A12) $S'_{a,t} = Y^M_{a,t} + G^{A'}_{a,t} + N^{D'}_{a,t} + N^R_{a,t} + N^{P\prime}_{a,t} - C_{a,t} = Y^{F'}_{a,t} + N^R_{a,t} + N^{P\prime}_{a,t} - C_{a,t} = \mathrm{S}_{a,t} + T^A_{a,t}$

Current corrected private savings is higher than private savings because contributions to social security are considered mandatory savings. $N^{P\prime}_{a,t}$ are private transfers when pensions are assumed to be deferred income. However, after the current period, the relationship between forced savings and contributory pension payments must be adjusted for the reasons that we have already explained, associated with the fact that, from an intertemporal perspective, the government has a *de facto* liability that declines over time and the private sector should finance an increasing part of private pensions. Consequently, the cohort's a savings ratio with pensions as deferred income will be:

(13) $\sigma'_{a,t} = r\,\omega^p_{a,t-1}\,\frac{(1+x_t)}{(1+g)(1+x_{a,t})} + \eta^{D'}_{a,t} + \frac{G^{A\prime}_{a,t}}{X_{a,t}y_t} + \eta^R_{a,t} + \eta^{P\prime}_{a,t} - \delta_{a,t} = \sigma_{a,t} + \frac{T^A_{a,t}}{X_{a,t}y_t}$

Wealth evolves according to:

(A14) $\omega^{p'}_{a_0,N} = \left(1+\lambda)^N\omega^{p'}_{a_0,0} + \sum_{t=1}^{N}(1+\lambda)^{N-t}(\eta^{D'}_{a_0,t} + \frac{G^{A'}_{a,t}}{X_{a,t}y_t} + \eta^R_{a_0,t} + \eta^{P'}_{a_0,t} - \delta_{a_0,t} + \frac{T^A_{a_0,t}}{X_{a_0,t}\,y_t}\right)$

And the life-cycle demand for wealth should be expressed as:

(A15) $\pi_{a_0,0} = \omega^p_{a_0,0} + \sum_{t=1}^{\bar{a}-a_0}(1+\lambda)^{-t}\frac{T^A_{a_0t}}{X_{a_0,t}y_t} + \sum_{t=1}^{\bar{a}-a_0}(1+\lambda)^{-t}(\eta^{D\prime}_{a_0,t} + \frac{G^{A\prime}_{a_0t}}{X_{a_0,t}y_t} + \eta^R_{a_0,t} + \eta^{P\prime}_{a_0,t})$

Where $\eta^{D\prime}_{a_0,t}$ and $\eta^{P\prime}_{a_0,t}$ stands for government redistributions and private transfers adjusted to reflect the changes in the assumptions concerning the social security system. If the period of duration a_0 covers the entire period during which the a_0 cohort accumulated "forced savings" and received pension payments and payments were "fair", the pension payments that the a_0 cohort should receive should be in the amount of:

(A16) $\sum_{t=-a_0}^{t=0}(1+\lambda)^{-t}\frac{T^A_{a_0,t}-G^{A'}_{a_0,t}}{X_{a_0,t}y_t} = \sum_{t=1}^{\bar{a}-a_0}(1+\lambda)^{-t}\frac{G^{A'}_{a_0,t}}{X_{a_0,t}y_t}$

Three points deserve highlighting. First, the total amount of the subsidy in present value at time *t=1* can be calculated as:

(A17) $\frac{Z^{A'}_{a_0,1}}{X_{a_0,1}\,y_1} = \sum_{t=1}^{\bar{a}-a_0}(1+\lambda)^{-t}\frac{G^{A'}_{a_0t}}{X_{a_0,t}y_t} - \sum_{t=a_{0,}}^{t=0}(1+\lambda)^{-t}\frac{T^A_{a_0,t}-G^{A'}_{a_0,t}}{X_{a_0,t}y_t}$

Obviously, if $\frac{Z^{A'}_{a_0,1}}{X_{a_0,1}\,y_t}$ <0, it will be a tax rather than a subsidy. Second, if *r > g*, and the public sector pays the market rate of return on forced savings, it follows that pension payments will absorb a growing amount of per capita income because, *ceteris paribus*, the demand for life-cycle wealth increases with λ. This will tend to crowd out other fiscal redistributions or create the need to increase the tax burden. Third, we are assuming that $\bar{a}$ is constant. However, if $\bar{a}$ increases because of an increase in longevity of the kind that is being observed in advanced economies, the probability that the social security system subsidizes future pensioners will be higher.

To illustrate the linkage between demography and wealth distribution, consider an extreme case in which cohort a_0 is comprised of two groups (*d=h,l*) – the wealthy who own all private wealth ($a_{0,h}$) and are formal workers, on the one hand, and the poor ($a_{0,p}$) who own no wealth at all and participate in the informal labor market because of a lack of human capital, on the other. If we consider that $T^A_{a_0,t}$ are forced savings, from (67) it follows that the demand for life-cycle wealth of the wealthy will be:

(A18) $\pi_{a_0,0,h} = \omega^P_{a_0,0,h} + \sum_{t=1}^{\bar{a}-a_0}(1+\lambda)^{-t}\frac{T^A_{a_0,t,h}}{x_{a_0,t,h}\,y_t} + \sum_{t=1}^{\bar{a}-a_0}(1+\lambda)^{-t}(\eta^{D\prime}_{a_0,t,h} + \frac{G^{A\prime}_{a_0,t,h}}{x_{a_0,t,h}\,y_t} + \eta^R_{a_0,t,h} + \eta^{P\prime}_{a_0,t,h})$

and for the poor:

(A19) $\pi_{a_0,0,l} = \sum_{t=1}^{\bar{a}-a_0}(1+\lambda)^{-t}(\eta^D_{a_0,t,l} + \eta^R_{a_0,t,l} + \eta^P_{a_0,t,l})$

Under these circumstances, the poor would depend entirely on public and private transfers to finance their demand for life-cycle wealth. But the bequests received by the poor in the form of assets and financing for the accumulation of human capital will probably be very low. Consequently, the members of $a_{0,l}$ could easily get caught in a poverty trap. They do not participate in the formal labor market because of the lack of human capital and they cannot accumulate human capital and forced savings because they do not participate in the formal labor market and, as a consequence, their income is insufficient. If the coverage of the social security system were reduced or biased in favor of the richer — whose participation in the formal labor market is higher — the prospects for those elderly that are also poor would be discouraging. Forced contributions would accumulate in the pension accounts of the wealthy, increasing formal workers' wealth ($\omega^P_{a_0,t,h}$) over time. Furthermore, if *r > g* the benefits of forced savings will increase at a higher path than income. The higher the difference between *r* and *g*, the lower will be the present value of the life-cycle wealth of the poor ($\pi_{a_0,0,l}$), while the opposite case will hold for the wealthy because, obviously, the value of $\omega_{a_0,0,h,}$will not be affected.

But even if the lowest strata have some wealth, if the poor's birth rate is higher, given the return on capital, the rate of growth of their per capita wealth will be lower (because the effective rate λ' would be lower. Remember that we are assuming λ=λ^a to simplify). This factor will also favor the generation of poverty traps. Indeed, if $(\eta^{D\prime}_{a_0,t,h} + \eta^R_{a_0,t,h}) < 0$, a lower present value will favor the wealthy.

Annex 2

Nomenclature

a Subscript that identifies the cohort
$'$ Superscript that states that pensions are considered deferred incomes
$\bar{a}$ Oldest cohort
B_t^G Stock of public debt (net)
b_t Government bonds per capita
$\tilde{b}_t^G = \frac{B_t^G}{Y_t}$
B_t^P Public bonds held by the private sector
$\tilde{b}_t^P = \frac{B_t^P}{Y_t}$
$C_{a,t}$ Consumption of cohort a
$c_{a,t}$ Cohort a's per capita consumption
C_t^E Number of effective consumers in cohort a
d Subscript that identifies the strata
$\bar{d}$ Highest strata
D_t^G Fiscal deficit

D_t^{PG} *Primary fiscal deficit*
$\tilde{d}_t^G = \frac{D_t^G}{Y_t}$
$\tilde{d}_t^{PG} = \frac{D_t^{PG}}{Y_t}$
E_t^G *Total rents from natural resources*
$\bar{E}_t^G$ *Deduction for depletion of natural resources*
$f_{a,d,t}$ *Employment intensity*
g *Growth rate of GDP*
g^n *Nominal growth rate of GDP*
G_t^A *Pension transfers*
G_t^E *Expenditures in kind related to education*
G_t^H *Expenditures in kind related to health*
G_t^I *Indirect subsidies*
G_t^R *Other public goods*
G_t^O *Other transfers*
h *High income*
I_t^G *Government investment*
K_t^G *Government's stock of capital*
K_t^P *Private stock of capital*
l *Low income*
L_t *Number of effective workers*
$L_{a,t}$ *Number of effective workers of the a cohort*
$lcd_{a,t}$ *Per capita life-cycle deficit of the a cohort*
LCD_t *Life cycle deficit*
m_t *Proportion of natural resource depletion substracted*
N_t^D *Fiscal redistributions*
n_t^D *Per capita Fiscal redistributions per capita*
$N_{a,t}^P$ *Net transfers received by cohort a from other cohorts*
N_t^R *Net incidence of other budget items*
p_t *Natural resource rents*
$\hat{p}_t$ *Rate of growth of rents*
Q_t^G *Government-owned reserves of natural resources*
q_t^G *Per capita government-owned reserves of natural resources*
r *Interest rate*
$S_{a,t}$ *Savings of the a cohort*
S_t^G *Net government savings*
s_t^G *Per capita net government savings*
S_t^P *Private savings*
$S_{a,t}$ *Cohort a savings*
SR_t *Support ratio*
$SR_{a,t}$ *Cohort a support ratio*
T_t^A *Employee contributions to social security*
T_t^F *Fees*
T_t^I *Indirect taxes*
T_t^R *Other revenues (including corporate taxes)*
T_t^Y *Personal taxes*
$u_{a,t} = \frac{X_{a,t}}{X_t}$
$v_{a,t}$ *Average wage*
$\tilde{v}_{a,d,t}$ *Wage correction factor corresponding to group a,d*
W_t^G *Government wealth*

w_t^G *Per capita government wealth*
W_t^P *Private wealth*
$W_{a_0,t}^p$ *Wealth at time t of the cohort that was a_0 years old at time t=0*
$w_{a,t}^p$ *Per capita wealth*
X_t *Total population*
$X_{a,t}$ *Total population of cohort a*
x_t *Growth rate of the total population*
$x_{a,t}$ *Rate of growth of the population of age a*
$X_{a,d,t}^e$ *Members of the group that are employed*
Y_t *GDP*
y_t *Per capita GDP*
Y_t^A *Income from accumulated assets*
$Y_{a,t}^A$ *Cohort a income from accumulated assets*
$y_{a,t}^A$ *Cohort a per capita income from accumulated assets*
Y_t^C *Consumable income*
Y_t^D *Disposable income*
Y_t^F *Final income*
y_t^F *Per capita final income*
Y_t^L *Labor income*
$y_{a,t}^L$ *Cohort a per capita labor income*
Y_t^M *Market income*
z_t *"Extra consumption" financed by capital gains*
Z_t^A *Subsidized part of contributory pensions*
α_t *Benefits received from the government*
$\alpha_{a,t} = \frac{G_{a,t}^O+G_{a,t}^A+G_{a,t}^I+G_{a,t}^E+G_{a,t}^H+G_{a,t}^R}{X_{a,t}y_t}$
$\alpha_t^D = \frac{G_t^O+G_t^A+G_t^I+G_t^E+G_t^H}{Y_t}$
$\alpha_t^R = \frac{G_t^R}{Y_t}$
β_t *Tax burden*
$\beta_{a,t} = \frac{T_{a,t}^A+T_{a,t}^Y+T_{a,t}^I+T_{a,t}^F+T_{a,t}^R}{X_{a,t}y_t}$
$\beta_t^D = \frac{T_t^A+T_t^Y+T_t^I+T_t^F}{Y_t}$
$\beta_t^R = \frac{T_t^R}{Y_t}$
γ_t *Aggregate Labor share*
$\gamma_{a,t} = \frac{y_{a,t}^L}{y_t}$
$\delta_t = \frac{LCD_t}{Y_t}$
$\delta_{a,t} = \frac{lcd_{a,t}}{y_t}$
$\varepsilon_t^G = \frac{E_t^G}{Y_t}$
$\bar{\varepsilon}_t^G = \frac{\bar{E}_t^G}{Y_t}$
ε_t^p *Non-labor income share*
$\varepsilon_{a,t}^p$ *Cohort a non-labor income share*
ς *Depreciation rate of the capital stock*

$\eta_t^D = \frac{N_t^D}{Y_t}$

$\eta_t^R = \frac{N_t^R}{Y_t}$

$\iota_t^G = \frac{I_t^G}{Y_t}$

$\kappa_t^G = \frac{K_t^G}{Y_t}$

$\kappa_t^P = \frac{k_t^P}{y_t}$

λ *Effective interest rate*

λ^a *Effective interest rate with* $x_t \neq x_{a,t}$

λ^* *Effective rate in terms of* $\hat{p}_t$

$\xi_t^G = \frac{Q_t^G}{Y_t}$

π *Life cycle wealth*

$\sigma_t^G = \frac{S_t^G}{Y_t}$

$\sigma_t^P = \frac{S_t^G}{Y_t}$

$\sigma_{a,t} = \frac{s_{a,t}}{y_t}$

$\varphi_{a,t} = \frac{c_{a,t}}{y_t}$

φ_t *Aggregate propensity to consume*

$\psi_t^F = \frac{y_t^F}{y_t}$

$\psi_t^M = \frac{y_t^M}{y_t}$

$\omega_t^G = \frac{W_t^G}{Y_t}$

$\omega_t^P = \frac{W_t^P}{Y_t}$

$\omega_{a,t}^p = \frac{w_{a,t}^p}{y_t}$

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